Jurnal Math Educator Nusantara
Wahana publikasi karya tulis ilmiah di bidang pendidikan matematika
p-issn: 2459-9735 e-issn: 2580-9210
http://ojs.unpkediri.ac.id/index.php/matematika

Designing arithmetic sequence learning materials through the PMRI approach in the gelora sriwijaya stadium context

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Article received : December 10, 2025.
Article revised : February 18, 2026.
Article Accepted : February 26, 2026.
Article Publish : May 20, 2026
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Abstract: Arithmetic sequences are an important topic in mathematics that requires conceptual understanding. However, in classroom practice, this material is often taught without meaningful real-world contexts, leading students to memorize formulas rather than understand underlying concepts. Therefore, this study aimed to design arithmetic sequence learning using the PMRI approach in the context of the Gelora Sriwijaya Stadium. This study employed qualitative design research consisting of three stages: preliminary design, design experiment (pilot and teaching experiment), and retrospective analysis. The participants were 34 eleventh-grade students at SMA Negeri 12 Palembang, South Sumatra, Indonesia. Data were collected through classroom observations, students' activity sheets (LKPD), and semi-structured interviews, and analyzed descriptively by comparing the initial Hypothetical Learning Trajectory (HLT) with the actual learning trajectory. The findings show that the stadium context supported students in progressively constructing the concept of arithmetic sequences, moving from concrete representations of seating arrangements to identifying constant differences and formulating the nth-term expression through guided reinvention. The activities promoted self-generated models that bridged informal reasoning and formal mathematical representation. This research produced learning design for arithmetic sequence material based on real contexts, which serves as an alternative for teachers in implementing PMRI.
Keywords: arithmetic sequence; gelora sriwijaya stadium; PMRI approach; learning design

Desain pembelajaran materi barisan aritmatika dengan menggunakan pendekatan PMRI pada konteks stadion gelora sriwijaya

Abstrak: Barisan aritmatika menjadi salah satu topik penting dalam matematika yang membutuhkan pemahaman konsep dalam mempelajarinya. Namun kenyataannya, materi ini diajarkan tanpa dikaitkan dengan keadaan nyata dan diterapkan dengan menggunakan model, metode, teknik, ataupun strategi yang kurang tepat, sehingga peserta didik hanya menghafal rumus tanpa memahami konsep tersebut. Oleh karena itu, tujuan dari penelitian ini adalah mendesain pembelajaran materi barisan aritmatika dengan menggunakan pendekatan PMRI pada konteks stadion Gelora Sriwijaya. Stadion Gelora Sriwijaya menjadi konteks yang tepat dalam memahami konsep barisan aritmatika. Penelitian ini menggunakan metode Design Research yang terdiri perancangan awal, eksperimen desain, dan analisis retrospektif. Data dikumpulkan melalui lembar aktivitas siswa dan wawancara. Subjek yang digunakan adalah peserta didik kelas XI SMA yang berjumlah 34 orang. Hasil penelitian menunjukkan bahwa konteks stadion Gelora Sriwijaya membantu siswa dalam membangun pemahaman konsep barisan aritmatika secara bertahap, dimulai dari kegiatan konkret mengamati susunan kursi stadion, menyusun objek potongan kertas membentuk sketsa kursi stadion, menentukan pola dan beda antar barisan, hingga akhirnya mampu menggeneralisasi rumus barisan aritmatika secara formal. Penelitian ini menghasilkan desain pembelajaran materi barisan aritmatika berbasis konteks nyata yang menjadi alternatif bagi guru dalam menerapkan PMRI.

Kata Kunci: barisan aritmatika; stadion gelora sriwijaya; pendekatan PMRI; desain pembelajaran

CITATION FORMATS: Azzahroh, F., Zulkardi, Z., Susanti, E., & Meryansumayeka, M. (2026). Designing Learning Materials for Arithmetic Sequence Using the PMRI Approach in the Context of Gelora Sriwijaya Stadium. *Jurnal Math Educator Nusantara: Wahana Publikasi Karya Tulis Ilmiah Di Bidang Pendidikan Matematika*. 12(1), 79-98. https://doi.org/10.29407/jmen.v12i1.27677

INTRODUCTION

Arithmetic sequences are an important topic in high school mathematics. This topic not only serves as a foundation for further discussion of other mathematical topics, but also has strong connections to various situations in everyday life. Arithmetic sequences are a topic that is often used in various aspects of daily life (Jupri et al., 2022). In addition, students are expected to be able to apply and solve problems related to mathematics in their daily lives (Sharma, 2021). This implies that students should be capable of connect real-world situations with mathematical concepts. Therefore, arithmetic sequences can be linked to and used as a solution in solving problems in everyday life (Nurwahid & Ashar, 2022).

However, in reality, students' proficiency in comprehending mathematical concepts in arithmetic sequences is still low (Irawati et al., 2022). This can be seen from the difficulty students have in using formulas to solve story problems or contextual problems, because they only memorize the formulas (Swastika et al., 2022). This is consistent with the findings of the study conducted by Kurniasari et al., (2022), that many students continue to experience errors when working on problems because they rely on memorizing formulas rather than truly understanding the components of arithmetic sequences and series. Moreover, learners often struggle to derive the formula fot the n term in arithmetic sequence tasks and have not fully grasped the idea of the sequence initial term (Hardiyanti, 2016).

Preliminary observations conducted at the research site support these findings. Based on an interview with the mathematics teacher, many students experience difficulties in identifying patterns and explaining their reasoning when solving arithmetic sequence problems. The teacher also noted that students frequently apply formulas mechanically without understanding their meaning, which results in errors when dealing with contextual questions. These initial findings indicate that the problem of low conceptual understanding is not only reported in previous studies but is also evident in the school where this research was conducted. Therefore, strengthening students' conceptual understanding of arithmetic sequences is necessary so that they can solve problems more meaningfully and accurately.

The reason why students have difficulty solving arithmetic sequence problems is because they do not yet understand the concept of sequences (Almaghfiroh & Mukti, 2022). In learning, the teaching materials used by teachers are often irrelevant to the lives of students, making it difficult for them to understand mathematical concepts, especially arithmetic sequences (Fitzmaurice et al., 2021). In addition, the application of models, methods, techniques, or strategies greatly influences the instilling of mathematical concepts in students in order to meet the intended learning outcome (Agbata et al., 2024).

One of the activities that supports student learning progress in arithmetic sequences is the Indonesian Realistic Mathematics Education (PMRI/RME) approach (Sutiarso, 2023). PMRI is a form of learning that uses the real world and learning activities that emphasize student activities to search for, discover, and build the necessary knowledge themselves so that learning is student-centered (Pebriana, 2017). PMRI is one approach that uses contextualization (Rawani & Octaria, 2023). In addition, the selection of learning contexts and media must align with the mathematical ideas being taught, as the chosen contexts embed these concepts in a way that is readily accessible to students (imagine) and recognize (Yilmaz, 2020). Contextual situations can serve as an initial basis to bridge students' activities in understanding mathematical concepts (Sevinc & Lesh, 2022). Context plays a major role at the beginning of learning and is presented present as problem tasks (Sari & Noviartati, 2022). The selection the context selected must fit the mathematical concept under study, since it conveys the mathematical ideas in a form students can easily interpretaion concepts in a form that is easily understood by students and recognize (Cai & Hwang, 2023).

Previous studies have implemented the PMRI approach using cultural building contexts, such as the limasan house by (Marella & Fiangga, 2024), Palembang songket fabric by (Indriani et al., 2018), and buna woven fabric by (Maifa et al., 2020). In the studies by (Marella & Fiangga, 2024; Indriani et al., 2018), cultural patterns from traditional architectural elements and songket motifs were used as entry points for students to identify mathematical regularities and construct sequence concepts. Although these contexts incorporate rich local culture, the relationship between the visual patterns and the arithmetic sequence structure is indirect; students first need to abstract the visual or decorative patterns into numbers before identifying the constant difference between sequence terms. The study by Maifa et al. (2020) used cultural artifacts for geometry learning but did not focus on numerical pattern growth as represented in arithmetic sequences. Therefore, while these cultural contexts are valuable for introducing mathematical ideas, their connection to the core structure of arithmetic sequences is not as explicit or immediately observable as needed for systematic mathematization.

In contrast, the present study focuses more specifically on supporting students' conceptual understanding of arithmetic sequences through a context that structurally represents consistent numerical growth. The Gelora Sriwijaya Stadium was selected not only because it is familiar to students in South Sumatra, but also because the arrangement of its seats provides a clear and scalable model of increasing patterns that can be directly linked to arithmetic sequence concepts. Each row of seats increases by a constant amount, making the idea of numerical progression and difference between consecutive elements directly observable. Compared to woven motifs or architectural patterns, the stadium seating arrangement can be visualized more explicitly as a linear numerical model, making it easier for students to move systematically from concrete observation to mathematical generalization. As a context closely related to students' real experiences, the stadium helps students connect mathematical ideas to meaningful situations (Harefa, 2024), and recognize arithmetic sequences as patterns embedded in real-life structures (Siregar, 2025).

This research was conducted to design arithmetic sequence learning using the PMRI approach in the context of the Gelora Sriwijaya Stadium. The learning design was developed using a Hypothetical Learning Trajectory (HLT) (Saefudin et al., 2023). HLT describes the intended learning goals, the sequence of instructional activities, and predictions about how students' thinking is expected to develop throughout the learning process (Lantakay et al., 2023). Based on this framework, this study aims to develop a coherent learning design that supports students in constructing a deeper understanding of arithmetic sequences.

METHOD

The research method used in this study was design research based on the Gravemeijer and Cobb model, which consists of three main stages: preliminary design, design experiment, and retrospective analysis (Yuliasari & Arnawa, 2021). The main objective of this study was to design learning for the topic of arithmetic sequences using the PMRI approach and the context of the Gelora Sriwijaya stadium. The learning path was designed in the form of learning activities that predicted students' strategies and responses in gradually building conceptual understanding, namely from the model of to the model for or from informal knowledge to formal knowledge. The research subjects consisted of 34 grade XI students at a high school in Palembang, South Sumatra.

In the preliminary design stage, the researchers conducted a literature review to identify materials that students found difficult to understand and explored appropriate alternative teaching approaches. The review indicated that arithmetic sequences are among the topics that students struggle with, as they frequently make errors when solving related problems. Arithmetic sequences are also appropriate for grade XI students (Qolbi et al., 2022). Therefore, the researchers designed student activity sheets (LKPD) based on PMRI principles linked to the context of the Gelora Sriwijaya Stadium.

The three learning activities consisted of observing and describing the seating pattern of the Gelora Sriwijaya Stadium, determining the constant difference in the number of seats between rows, and generalizing the relationship between terms to derive the nth-term formula. These activities were organized into a Hypothetical Learning Trajectory (HLT), which included learning objectives, planned student activities, and predictions of students' strategies and responses.

Before being implemented in the pilot experiment, the research instruments including the LKPD and the semi-structured interview guidelines underwent a validation process to ensure their content validity and clarity. The validation was carried out by mathematics education lecturers with expertise in PMRI, experienced mathematics teachers, and researchers. The validators examined the alignment between the tasks and the learning objectives, the suitability of the PMRI principles, the correctness of mathematical content, and the clarity of language and instructions. The interview guidelines were also reviewed to ensure that the questions were relevant to exploring students' conceptual understanding and were understandable for grade XI students. The validation results were in the form of suggestions and revisions, which were used to refine the LKPD, HLT, and interview protocol before they were tested in a small group.

The second stage was a pilot experiment aimed at assessing and refining the learning path that had been designed. At this stage, the HLT and LKPD that had been created were tested on six students who were not included in the research subjects, with the researcher acting as the model teacher. The trial was conducted in one session lasting 2×45 minutes, during which the researcher observed the learning process while providing facilitation as needed. The LKPD was used as a source of data to see how students completed the LKPD that was given, and the results became the basis for analyzing and improving the HLT before it was applied in the next stage, namely the teaching experiment involving 34 students. The groups were formed heterogeneously with seven groups to create interaction within the groups so that the learning objectives could be optimized (Klang et al., 2021).

Data were collected through classroom observations, students' activity sheets, and semi-structured interviews. The data were analyzed qualitatively using descriptive analysis and retrospective analysis by comparing the initial Hypothetical Learning Trajectory (HLT) with the actual learning trajectory that emerged during the teaching experiment. To ensure the trustworthiness of the findings, triangulation techniques were employed. Methodological triangulation was conducted by comparing data obtained from different collection methods, namely observations, students' written work, and interview results. In addition, data source triangulation was carried out by examining patterns of responses across different students to identify consistency and convergence of findings. The triangulation process strengthened the credibility of the analysis and minimized potential bias. The results of the retrospective analysis were subsequently used to refine the initial HLT and to formulate a Local Instructional Theory (LIT) for teaching arithmetic sequences within the context of the Gelora Sriwijaya Stadium.

RESULTS AND DISCUSSION

Preliminary design

The first step taken at this stage was to conduct an exploration through a literature review related to mathematical material that is difficult for students to understand, as well as an analysis of the applicable curriculum. This study aimed to build a strong theoretical foundation for the learning design. The results of the literature review indicated that arithmetic sequences are considered difficult by students, particularly when they rely on memorizing formulas without conceptual understanding. The material is appropriate for grade XI students, and the Gelora Sriwijaya Stadium was selected as a context due to its proximity and relevance to students' real experiences.

The selection of this context was not merely based on familiarity but also aligned with key PMRI principles. In particular, the stadium seating arrangement was chosen because it enables students to engage in guided reinvention, where mathematical concepts are reconstructed through meaningful contextual situations. Furthermore, the structured growth of the seating rows allows students to develop self-generated models, beginning from concrete representations (arranging paper squares) toward more formal numerical generalizations.

Researchers collaborated with mathematics teachers and mathematics education lecturers to develop the research design. Discussions focused on how the Gelora Sriwijaya Stadium context could meaningfully support progressive mathematization—from informal observation of seat patterns to formal reasoning about arithmetic sequences. At this stage, the Hypothetical Learning Trajectory (HLT) was developed in accordance with PMRI principles, anticipating how students might move from model-of the seating arrangement to model-for reasoning about numerical patterns. The HLT design plan is presented in Table 1.

Table 1. HLT Design

Objectives	Activities	Students' Conjectures
Students can find the formula for an arithmetic sequence through the context of the Gelora Sriwijaya Stadium.	Activity 1: Observe and describe the seating arrangement in the stadium 1) Students observe the picture of the stadium. 2) Students answer questions related to the picture they have observed. 3) Students read the context “circular stands” and predict the number of seats in a certain row. 4) Students illustrate the arrangement of stadium seats by arranging the available pieces of paper	1) Students can immediately see and recognize the growth in the seat count in each row. 2) Students only describe the shape of the stands without understanding the increase in the number of seats. 3) Students miscount the number of seats and assume that each row contains an identical number of seats.
	Activity 2: Determining patterns and differences (b) between rows 1) Students create a table containing the row index (n) and the total seats in that row in each row $(U_1, U_2, U_3, \ldots)$. 2) Students calculate the difference between two consecutive rows. 3) Students identify the difference (b) and represent it in symbolic form.	1) Students find the constant difference and conclude that there is a regular pattern of increase. 2) Students can determine $\mathbf{U_1}, \mathbf{U_2}, \mathbf{U_3}$, but have not realized that the difference remains. 3) Students are confused when calculating differences or make mistakes when recording data in tables.
	Activity 3: Generalizing the formula for the term (U_n). 1) Students can explain how to obtain U_2 and U_3 from U_1 by adding the difference (b). 2) Students are able to develop general patterns in U_1, U_2, U_3, U_4.	1) Students understand the pattern and can immediately formulate the formula U_n. 2) Students recognize the increase in b but have difficulty connecting $(n-1)$ with row n. 3) Students write the formula incorrectly.

	3) Students can form the n term formula, which is $U_n = a + (n-1)b$. 4) Students can determine how many seats are in the row 25 using the formula they have discovered.	

Subsequently, the LKPD was verified through expert review involving researchers, mathematics teachers, and mathematics education lecturers before being used in a pilot experiment. The verification process focused on several aspects, namely content validity (suitability with learning objectives and mathematical concept accuracy), construct validity (suitability of tasks with the PMRI and HLT principles that had been designed), clarity of language, and learning design. The verification results showed that the LKPD met the validity criteria with minor revisions. The suggested revisions included improving the instructions so as not to cause confusion among students and refining the construct aspect so that the sequence of activities was clearer and helped students gradually understand and construct U_n the formula. The comments and suggestions provided is presented in Table 2 below.

Table 2. Comments and Suggestions on the LKPD

Comments and Suggestions	Revision Results
From the students' activities, the main points should be emphasized more so that students understand the differences, and the role of teachers as facilitators is to guide students towards the U_n formula.	Before: • Apakah jumlah kursi tiap baris selalu sama? Jika tidak, tuliskan selisih jumlah kursi antar barisan! • Apakah selisih ini tetap atau berubah? Pola apa yang terbentuk? After: 1 Perhatikan gambar susunan kursi pada setiap baris. Buatlah tabel yang berisi : nomor barisan (n) dan jumlah kursi pada setiap baris! 2 Apakah jumlah kursi tiap baris selalu sama? Jika tidak, hitunglah dan tuliskan selisih jumlah kursi dua baris yang berurutan! 3 Misalkan: Jumlah kursi pada baris ke-l adalah Ul. Jumlah kursi pada baris ke-2 adalah U2. Jumlah kursi pada baris ke-3 adalah U3, dan seterusnya Tuliskanlah nilai Ul, U2, dan U3 sesuai hasil pengamatanmu!
Provide instructions or add questions that guide students to the formula U_n, for example 14 + 14, which also means 2 x 14, and also $(n-1)$ is obtained from the n term minus 1.	Before: • Bagaimana jika ingin mencari jumlah kursi pada baris ke-n? Apakah menggunakan aturan tertentu? Jika iya, temukan rumus untuk menentukan jumlah kursi pada barisan ke-n. Jawab:

	After: 4 Bagaimana cara kamu mendapatkan U2 dari UI? Tambahkan apa agar hasilnya sama dengan U2? 5 Bagaimana cara kamu mendapatkan U3 dari UI? Tambahkan apa dan berapa kali? 6 Tuliskan hubungan yang kamu temukan! 7 Tuliskan pola yang kamu lihat seperti ini: UI = ... U2 = UI + ... U3 = UI + ... U4 = UI + ... 8 Perhatikan angka yang dikalikan dengan selisih (b). Apakah ada pola diantara angka tersebut dan urutan barisnya? 9 Jika kamu ingin mencari jumlah kursi di baris ke-n, berapa kali kamu harus menambahkan selisih (b) ke UI ? 10 Tuliskan idemu sendiri dalam bentuk kalimat! 11 Dari kalimatmu, ubahlah menjadi bentuk rumus sendiri (tanpa melihat contoh).

After revising the LKPD, the next activity was to conduct direct observations at the school that would be used as the experiment location to ensure that the design was in accordance with the characteristics of the students. This observation provides an overview of the initial abilities and learning patterns of the students who will be involved in the learning stage, as well as conducting interviews with mathematics teachers at the school. From the interview results, it was found that arithmetic sequences are taught in grade XI, which is in accordance with the results of the researcher's literature review.

Design Experiment

Table 3. Revision of the LKPD after the Pilot Experiment

Before revision	After revision
In question number 3, students had difficulty representing the answer in symbolic form. 3 Misalkan: Jumlah kursi pada baris ke-I adalah UI. Jumlah kursi pada baris ke-2 adalah U2. Jumlah kursi pada baris ke-3 adalah U3, dan seterusnya Tuliskanlah nilai UI, U2, dan U3 sesuai hasil pengamatanmu!	Instructions regarding the first term were added to guide students toward a regular pattern of increase. 3 Misalkan: Jumlah kursi pada baris ke-I adalah U_1 Jumlah kursi pada baris ke-2 adalah U_2 Jumlah kursi pada baris ke-3 adalah U_3, dan seterusnya Karena U_1 merupakan jumlah kursi pada baris ke-I, maka disimbolkan dengan "a" Tuliskanlah nilai U_1, U_2, dan U_3 sesuai hasil pengamatanmu!
In question number 7, some students did not realize that there was a regular pattern of increase. 7 Tuliskan pola yang kamu lihat seperti ini: UI = ... U2 = UI + ... U3 = UI + ... U4 = UI + ...	The symbols are clarified in detail to guide students in finding the U_n formula. 7 Tuliskan pola yang kamu lihat seperti ini: $U_1 = \cdots$ $U_2 = U_1 + \cdots$ $U_3 = U_1 + \cdots + \cdots$ $U_4 = U_1 + \cdots + \cdots + \cdots$

This phase consisted of a pilot experiment and a teaching experiment. During the pilot experiment, the learning design was tested with six students representing high, moderate, and low ability levels. The purpose was not only to evaluate clarity and difficulty level, but also to examine whether the learning activities genuinely facilitated guided reinvention as anticipated in the HLT. Findings from the pilot showed that clearer scaffolding was needed to support students in articulating the constant difference between rows. Revisions were therefore made to strengthen prompts that encourage students to express their reasoning explicitly rather than immediately resorting to formula use.

Based on the revised HLT and LKPD, both during the validation process and pilot experiment, the researcher will then test the design during the teaching experiment phase. In the teaching experiment involving 34 grade XI students, the learning trajectory was implemented in three stages. The lesson began by linking arithmetic sequences to the context of the Gelora Sriwijaya Stadium, particularly the orderly arrangement of seats. This contextual starting point functioned as a model-of the real situation. Through classroom discussion and group interaction, the seating arrangement gradually evolved into a model-for reasoning about regular numerical growth, reflecting the PMRI principle of progressive mathematization.

Activity 1: Observing and Describing the Stadium Seating Arrangement

In the initial activity, students examined an image and sketch of the stadium seating layout. They were then asked to arrange square pieces of paper, where one square represented seven seats. For example, the first row consisted of five squares, the second of seven squares, and the third of nine squares.

Figure 1. Students are invited to learn about the context of the Gelora Sriwijaya Stadium

Next, students are directed to look at the sketch of the stadium seating arrangement in the worksheet. They are asked to arrange the objects provided, which are pieces of paper, to form a sketch of the seating arrangement. The purpose of this activity is to build students' initial understanding through concrete visualization of the seating arrangement so that they can easily see the pattern of the rows in the stadium seats.

ISSN 2459-9735 (print), ISSN 2580-9210 (online)

Figure 2. Gelora Sriwijaya stadium sketch instructions

This hands-on activity was intentionally designed to encourage students to construct meaning through concrete manipulation. By physically arranging the squares, students generated their own representation of the seating pattern. This process exemplifies the PMRI principle of self-developed models, as students' arrangements served as bridges between the real context and mathematical reasoning. As students counted the total seats in each row and discussed how the number changed from one row to the next, they began to recognize the constant increase. Rather than being directly introduced to a formal formula, students were guided to rediscover the structural characteristic of arithmetic sequences through observation and discussion. This reflects the principle of guided reinvention, where students reconstruct the concept of constant difference through meaningful activity. Thus, the activity did not merely illustrate a pattern but functioned as a foundation for progressive mathematization from concrete seating arrangements to abstract numerical generalization.

Figure 3. Students visualize the seating arrangement of the Gelora Sriwijaya stadium

Activity 2: Determining Patterns and Differences (b) between rows

a. Students create a table showing the number of students and the number of seats in each row $(U_1, U_2, U_3, \ldots)$.

In the second activity, after the students arranged the LKPD instruction chair layout, they were asked to make a table containing the row number (n) and the number of chairs in each row. The students' answers is displayed in Figure 4.

ISSN 2459-9735 (print), ISSN 2580-9210 (online)

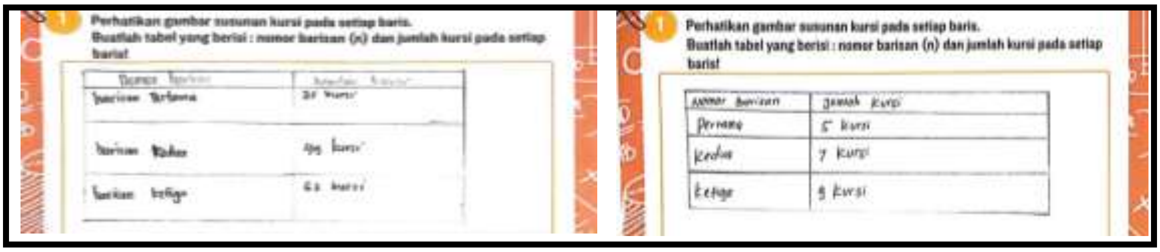

Figure 4. Students' answers at the table making stage

It can be seen that there is a difference in the answers between group (a) and group (b) in figuring out how many seats are in each row of the stadium. Group (a) listed the seat totals for the first, second, and third rows as 35, 49, and 63 seats. Meanwhile, group (b) wrote down 5, 7, and 9 seats, because they only counted the number of squares in the sketch of the Gelora Sriwijaya stadium and assumed that one square represented one seat. However, instructions had been given that 1 square represented 7 seats, so the correct answers should have been $5 \times 7 = 35, 7 \times 7 = 49$, and $9 \times 7 = 63$. Thus, the rows of seats should have formed an arithmetic sequence of 35, 49, and 63 with a constant difference of $\mathrm{b} = 14$, fulfilling the concept of a regular addition pattern. The difference in answers between groups (a) and (b) shows that group (b) did not understand that one square represents 7 seats, while group (a) calculated correctly and obtained the arithmetic sequence pattern of 35, 49, and 63. This is supported by interviews conducted during the learning process.

P *: How do you determine the number of seats in each row?*

G(a) *: We immediately counted the pieces of paper that had been arranged. For example, if there were 7 squares in the first row, that meant there were 35 seats in the first row, because 1 square represented 7 seats. The same applied to the other rows.*

P *: Group (b) how does it work?*

G(b) *: Our group immediately counted the seats, ma'am, because there are 5 seats in the first row, which means there are 5 seats. There are 7 seats in the second row and 9 seats in the third row, ma'am.*

P *: Has group (b) read the instructions carefully?*

G(b) *: Oh yes, ma'am, we didn't read it carefully when 1 block represents 7 seats. So we answered all the questions that haven't been converted to seats, ma'am.*

b. Students calculate the difference between two consecutive rows.

Figure 5. Students' answers in calculating the difference between two consecutive rows

In the second question in Activity 2, students are asked to observe the increase in the number of seats in each row and write down their observations. This activity can be seen from the answers of groups (a) and (b), where group (a) answered that the difference in the number of seats between two consecutive rows is 14 seats. This means that the number of seats in each row is not the same, and there is a difference in the number of seats between two consecutive rows that is always the same, namely 14 seats between the first and second rows, and 14 seats between the second and third rows. Meanwhile, group (b) answered 2 seats. This difference in answers arose because group (a) had converted the number of squares into seats according to the instructions, namely 1 square = 7 seats, resulting in a difference of 14 seats. Meanwhile, group (b) only compared the number of squares, namely $7 - 5 = 2$, without multiplying by the scale of 7, so the difference they obtained was the difference in squares, not the difference in seats.

c. Students identify the difference (b) and represent it in symbolic form.

Figure 6. Students' answers represented in symbolic form.

In this step, students are able to restate the seat count for every row by employing mathematical symbols so that the results of their observations regarding the number of seats in the first, second, and third rows are converted into more general mathematical notation, namely $\mathrm{U}_1, \mathrm{U}_2, \mathrm{U}_3$. By writing $\mathrm{U}_1 = a = 35, \mathrm{U}_2 = 49, \mathrm{U}_3 = 63$, students can see the pattern of increase between terms and calculate the difference. The use of symbols guides students in gradually discovering the formula for term Un, which is expected to provide students with understanding. However, in answer (b), it is written that $\mathrm{U}_1 = 5, \mathrm{U}_2 = 7, \mathrm{U}_3 = 9$, which means they can already generalize even though they have not yet converted from squares to seats.

Activity 3: Generalizing the n Term Formula (U_n)

After finding the pattern and differences between two consecutive rows, students are directed to explain how to obtain U_2 and U_3 from U_1, until finally writing down the general pattern. By comparing these patterns, students conclude how many times the difference is added to obtain U_n, then formulate an arithmetic the sequence formula to find the seat total in the n row. In the last step, learners use this formula to compute how many seats are in the 25 row as an application of the concept. Activity 3 consists of several questions that can guide students to derive the formula for the term in position n.

a. Students can explain how to obtain U_2 and U_3 from U_1 by adding the difference (b).

ISSN 2459-9735 (print), ISSN 2580-9210 (online)

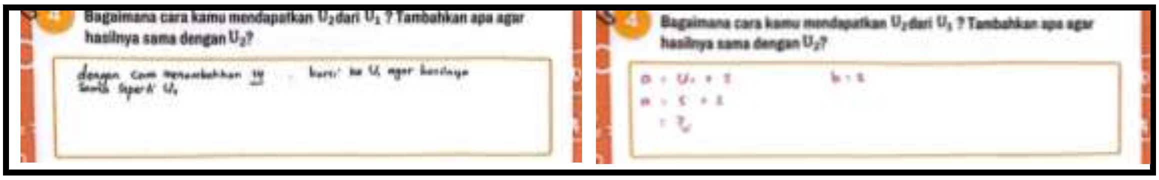

Figure 7. Students' answers in obtaining U_2 from U_1

Figure 7 shows that students were able to explain how to obtain U_2 from U_1. Group (a) explains it using words, while group (b) explains it using mathematical symbols. This means that groups (a) and (b) already understand that by adding the difference to U_1 will get the same result as U_2.

Figure 8. Students' answers in obtaining U_3 from U_1

In the next question, students were also asked to explain how to obtain U_3 from U_1 as can be seen in Figure 8. Groups (a) and (b), as well as other groups, were able to explain how to obtain U_3 from U_1. Students understand that U_3 can be obtained by repeatedly adding the difference, that is, adding the difference of the sequence once to obtain U_2, then adding it again to obtain U_3.

Afterward, students were instructed to record the relationships they found between $\mathrm{U}_1, \mathrm{U}_2, \mathrm{U}_3$, and b. Group (a) wrote that the relationship found was by adding 14 seats twice to get 63 seats or U_3. This means that group (a) understood that by repeatedly adding the difference, they could obtain each term. Meanwhile, group (b) wrote that the relationship they found was the difference between each sequence. However, they calculated the difference in terms of squares, not chairs. Figure 9 shows the students' answers in writing down the relationship between sequences.

Figure 9. Students' answers in finding the relationship between U_1, U_2, U_3, dan b

b. Students are able to develop general patterns $\mathrm{U}_1, \mathrm{U}_2, \mathrm{U}_3, \mathrm{U}_4$.

Figure 10 shows that students are already able to write down general patterns $\mathrm{U}_1, \mathrm{U}_2, \mathrm{U}_3, \mathrm{U}_4$ accurately. This means that they have demonstrated an understanding that each term is obtained from the sum of repeated differences. Mathematically, this observation means that students have recognized the structure of arithmetic sequences, namely that there is a constant difference (b) between terms, where the process is repeated addition of b, and the patterns from the pictures and tables have been successfully converted into more general notation. The ability to write patterns up to U_4 leads students to formal generalization, namely finding the formula for the nth term, so that students are not only able to calculate the next term one by one, but also find the nth term directly using the same concept.

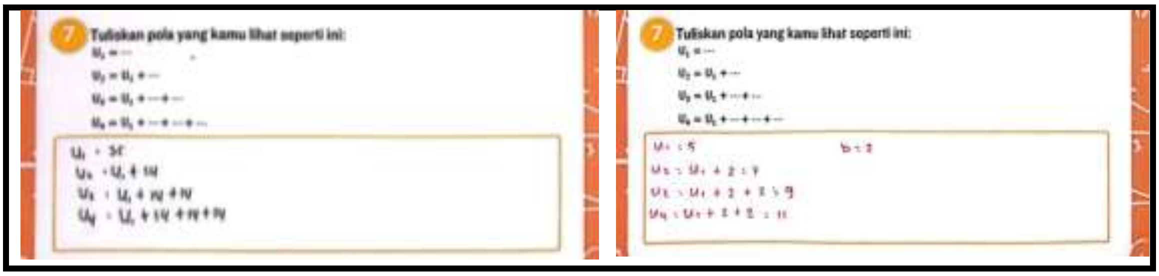

Figure 10. Students' answers form a general pattern

c. Students can form the n term formula, which is $U_n = a + (n-1)\,b$.

The next question before students form the formula U_n is that they must identify the pattern between the numbers multiplied by the difference (b) and the row sequence. This will lead students to discover $(n-1)$. In this question, group (a) wrote that as n increases, n is reduced by 1. Meanwhile, group (b) wrote how the pattern is formed using $b = 2$ and $U_n = 5$, so that the pattern can be seen. The following are the answers and interviews with the groups.

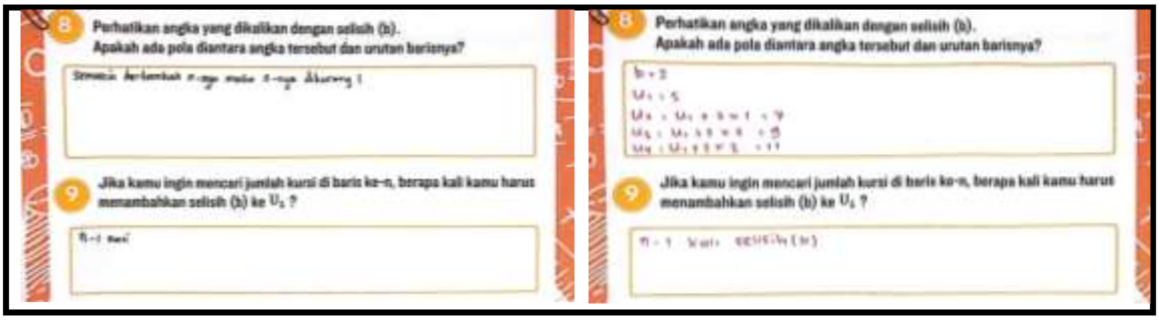

Figure 11. Students' answers in identifying patterns between the numbers multiplied by the difference and the row order

P : *n your opinion, what is the pattern between the numbers that are separated by the difference (b) and the row order?*

G1 : *Just as we wrote, the pattern is that as the value of n in U_n increases, the n multiplied by the difference will decrease 1.*

P : *For example, what does it look like?*

G1 : *Based on the question, we know that the difference is 14, and $U_1 = 35$. If we want to find U_2, then $n = 2$, which means $U_2 = 35 + (2-1)14$.*

P : *Group 2, how?*

G2 : *Our group wrote it like this, ma'am.*

$U_1 = 5$

$U_2 = U_1 + 2 \times 1 = 7$

$U_3 = U_1 + 2 \times 2 = 9$

$U_4 = U_1 + 2 \times 3 = 11$

So, if you want to find the n term, add $(n-1)$ *times the difference from* U_1.

G : *Okay, good.*

Based on the conversation, it can be concluded that students are already able to identify patterns formed from numbers multiplied by the difference (b) with the row order. Then, the next question asked students to form a formula from the results of pattern identification. It can be seen in Figure 12 that with several instructions that have been followed, students will be guided in forming the nth term formula. Students succeeded in writing the nth term formula correctly, namely $\mathrm{U_n} = \mathrm{a} + (\mathrm{n}-1)\,\mathrm{b}$.

Figure 12. Students' answers in forming the U_n formula

d. Students can calculate the number of seats in row 25 using the formula they found.

Figure 13. Students' answers in using the U_n formula

The final question asked students to use the formula they had discovered to determine the total number of seats in the 25 rows on one side of the stadium. Group (a) calculated correctly, while group (b) got a different answer. This was because from the outset, group b had misunderstood the number of seats in each section. At the end of the lesson, students concluded that the concept of arithmetic sequences can solve various contextual problems, including the arrangement of seats in the Gelora Sriwijaya stadium. This reinforces the meaning of learning mathematics as a means of understanding and managing real-life phenomena.

Retrospective analysis

After the design experiment stage was carried out, the next stage was retrospective analysis. Previously, it was seen that students began to model problems into mathematical

ISSN 2459-9735 (print), ISSN 2580-9210 (online)

forms through the horizontal mathematization process. At this stage, students were able to form the n term formula using mathematical symbols such as U_n, U_2, a and b. The development of students' thinking at this stage can be seen in Figure 14 below.

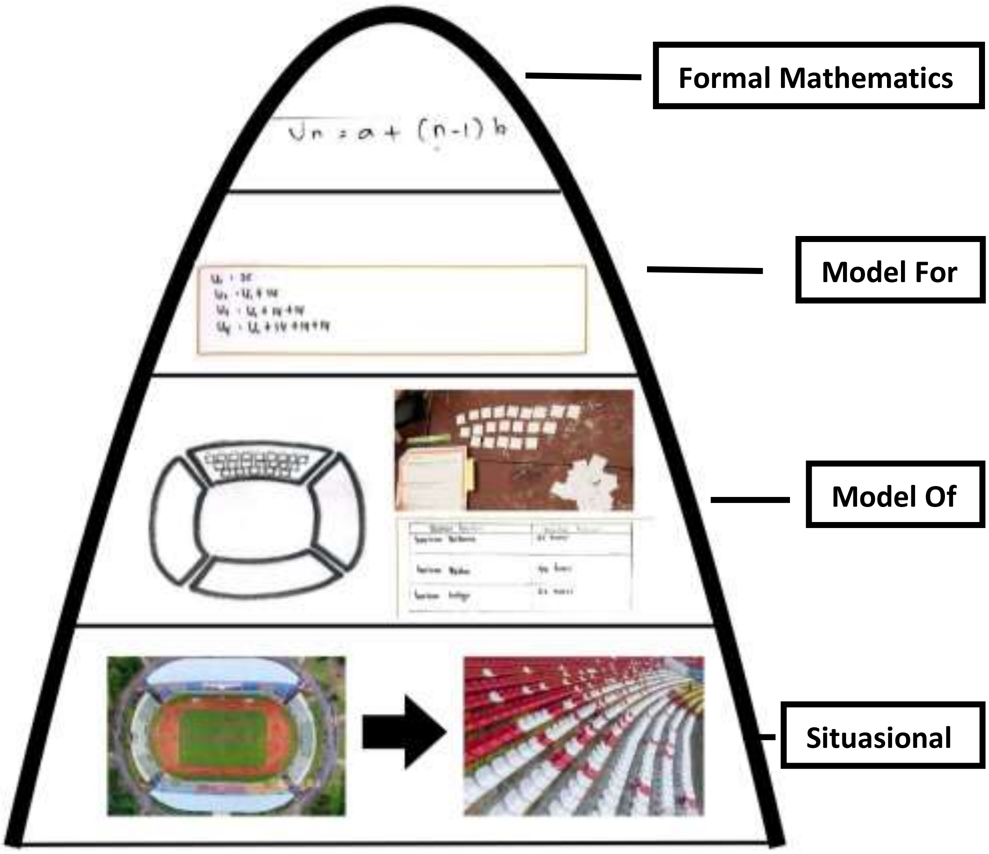

Figure 14. Iceberg obtained from retrospective analysis

Based on teaching experiments that have been conducted and retrospective analysis, it was found that the activities developed in HLT have been proven to assist students in grasping the idea of arithmetic sequences through the context of the Gelora Sriwijaya Stadium. This can be seen in Figure 14, which is an iceberg model of arithmetic sequences. This approach is structured so that students do not merely learn symbols and formulas, but also understand their meaning through real situations that are close to everyday life (Mangelep et al., 2023).

In the retrospective analysis phase, the researchers contrasted the initially planned learning trajectory (HLT) with the actual learning trajectory (ALT) that emerged during the pilot experiment and teaching experiment (Baroody et al., 2022). Analysis of student activity sheets and interview the findings indicated that the PMRI approach based learning design with the Gelora Sriwijaya stadium context generally went as expected. In the initial stage, students were able to arrange paper cutouts to form stadium seating and describe the number of seats in each row. However, some students still wrote the number of seats in squares without converting them to seats. This finding was in line with the HLT assumption that real-world contexts help build intuition about arithmetic sequences before students enter the mathematical model stage.

ISSN 2459-9735 (print), ISSN 2580-9210 (online)

At the model development stage (model of and model for), students demonstrate their ability to identify patterns in the increase in the number of seats between rows, calculate differences consistently, and represent relationships in symbolic form, such as U_n, U_2, a and b. In addition, students are able to generalize that to obtain the next term, they can add the difference repeatedly (Raviv et al., 2022). At this stage, the sketch of the stadium seats, which initially served as a model of, begins to shift to a model for, namely a mathematical thinking tool that helps students understand arithmetic sequences and build the structure of the relationship between terms.

At the formal stage, most students were able to determine that an arithmetic sequence referes to a list of numbers that has the same difference between two consecutive terms and were able to find the formula for arithmetic sequences, namely $U_n = a + (n - 1)$. This understanding does not arise instantly, but rather through a gradual process of concrete experiences, discussions, use of models, and symbolic abstraction, with teachers acting as facilitators who ask guiding questions (Gautam & Agarwal, 2023). This shows that students have reached the final stage in the learning process according to the iceberg model in PMRI.

Overall, the results of the retrospective analysis show that the learning design for arithmetic sequences using the context of the Gelora Sriwijaya stadium with the PMRI approach is effective in developing students understanding from informal to formal levels. This can be seen from the achievement of the assumptions in the HLT, most of which are in accordance with the ALT in the field (Baroody et al., 2022). Thus, the ALT that emerged in the classroom reinforces that the learning design is feasible and valid for use in teaching arithmetic sequences at the high school level.

CONCLUSION

The results of this study indicate that the learning design that uses the Gelora Sriwijaya Stadium context based on the PMRI approach is effective in helping students understand the concept of arithmetic sequences more deeply. The learning flow designed through HLT is in line with the development of students' thinking, starting from concrete activities such as observing the arrangement of stadium seats, arranging pieces of paper to form a sketch of the seats, identifying patterns and differences between numbers, and finally being able to formulate the general form of arithmetic sequences. The use of contexts familiar to students makes learning more meaningful because mathematical concepts are built gradually from real situations to more abstract forms.

However, this study is limited to the development of basic arithmetic sequence material and does not yet cover more complex material or application in more diverse problems. Therefore, further research is recommended to develop and test this learning design on more advanced material or other mathematical topics, as well as to explore the use of different contexts so that PMRI-based learning designs become more varied and comprehensive.

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