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Hypothetical learning trajectory of linear equation systems with three variables: The context of typical snacks riau islands

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Abstract: This study aims to design a learning trajectory for the Three-Variable Linear Equation System (SPLTV) by applying Indonesian Realistic Mathematics Education (PMRI) with the context of typical Riau Islands snacks. The research methodology involved three stages, namely preparing for the experiment, design experiment, and retrospective analysis. Data were collected through documentation and interviews, then analyzed by data reduction, data presentation, and conclusion drawing. The research subjects were 15 3rd semester students of the Mathematics Education Study Program at Riau Islands University. The results of the research are in the form of a hypothetical learning trajectory design in learning SPLTV with the context of typical snacks of the Riau Islands. The learning trajectory begins by linking students' learning experiences with the concepts to be presented. During learning, students apply the SPLTV concept to contextual problems. In the end, they are able to apply the SPLTV concept in different contexts, demonstrating an understanding of the material. This contextualized learning experience plays an important role in improving students' understanding of SPLTV.

Keywords: *Hyphotetical Learning Trajectory; Typical snacks of Riau Islands; Indonesian Realistic Mathematics Education; Linear Equation System of Three Variables*

Hypothetical Learning Trajectory Materi Sistem Persamaan Linear Tiga Variabel: Konteks Jajanan Khas Kepulauan Riau

Abstrak: Penelitian ini bertujuan mendesain lintasan belajar Sistem Persamaan Linear Tiga Variabel (SPLTV) dengan menerapkan Pendidikan Matematika Realistik Indonesia (PMRI) dengan konteks jajanan khas Kepulauan Riau. Metodologi penelitian melibatkan tiga tahap, yakni *preparing for the experiment, design experiment, dan retrospective analysis*. Data dikumpulkan melalui dokumentasi dan wawancara, lalu dianalisis dengan reduksi data, penyajian data, dan penarikan kesimpulan. Subjek penelitian adalah 15 mahasiswa semester 3 Program Studi Pendidikan Matematika Universitas Riau Kepulauan. Hasil penelitian berupa desain *hypothetical learning trajectory* dalam pembelajaran SPLTV dengan konteks jajanan khas kepulauan Riau. Lintasan belajar dimulai dengan mengaitkan pengalaman belajar mahasiswa dengan konsep yang akan disajikan. Selama pembelajaran, mahasiswa menerapkan konsep SPLTV pada permasalahan kontekstual. Pada akhirnya, mereka mampu mengaplikasikan konsep SPLTV dalam konteks yang berbeda, menunjukkan pemahaman materi. Pengalaman belajar kontekstual ini berperan penting dalam memperbaiki pemahaman mahasiswa terhadap SPLTV.

Kata Kunci: *Hyphotetical Learning Trajectory*; Jajanan Khas Kepulauan Riau; Pendidikan Matematika Realistik Indonesia; Sistem Persamaan Linear Tiga Variabel

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INTRODUCTION

One of the topics in math learning is the System of Linear Equations of Three Variables (SPLTV), which is generally presented in the form of story problems. This material was chosen because it is closely related to everyday life situations, such as calculating the total price of shopping without knowing the unit price of the items purchased, similar to the situation when we shop for several items but do not know the price per unit. Students who have difficulty in understanding mathematics often face challenges in calculating, solving story problems, or developing numeracy skills (Apsoh, et al, 2022).

Overcoming the challenges in learning the System of Linear Equations of Three Variables material, the Indonesian Realistic Mathematics Education approach emerges as a potential alternative solution. PMRI is a mathematics learning approach that begins the presentation of mathematical content by linking it to real situations that students are familiar with (Edwar, et al, 2023; Meryansumayeka, et al, 2022; Arnellis, et al, 2020; Aprilia & Awalia, 2020). In learning using PMRI, the reality and experiences of learners are placed as a starting point, providing a strong foundation for the learning process (Suyanti, et al, 2020). Previous research, such as the work of Fauziah & Putri (2017), Ekawati & Kohar (2016), and Meitrilova & Putri (2020), showed that students achieved better performance after learning with the PMRI approach. This approach provides hope for improving students' understanding of SPLTV material through the application of mathematical concepts in real contexts.

In this context, a learning trajectory is needed that can guide learners towards achieving the desired learning objectives. Learning trajectory, also known as Hypothetical Learning Trajectory (HLT), is a guideline that determines the activities that must be done in the learning process to achieve the expected learning objectives (Gee, 2019). HLT involves lecturers in making estimates of students' cognitive processes in a learning context. The components of HLT include learning objectives, learning activities, and hypotheses about how students' thinking and understanding will develop in the context of learning activities (Simon, 2014). Therefore, this research aims to produce a learning trajectory based on Indonesian Realistic Mathematics Education (PMRI) to facilitate the SPLTV learning process.

METHODS

This research is a Design Research that aims to improve the quality of learning by developing *Local Instructional Theory* (LIT) in economic mathematics courses on the topic of application of System of Linear Equations Three Variables (SPLTV) in economics, with its initial form in the form of *Hypothetical Learning Trajectory* (HLT). The research design consists of three stages, namely *preparing for the experiment, design experiment,* and *retrospective analysis, in* accordance with the model proposed by Akker et al. (2013).his process of designing learning trajectories is cyclical, creating a presumed local instructional theory. An explicit illustration of this research can be seen in Figure 1 below.

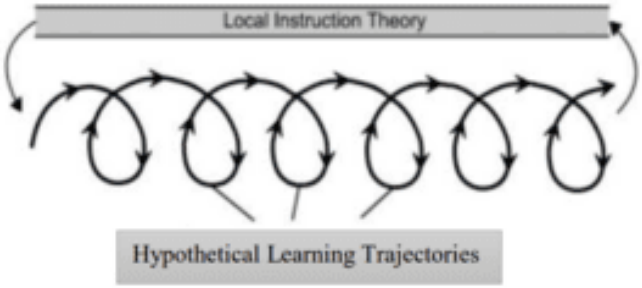


Figure 1: Cyclical Process

This research was conducted on 3rd semester students of the mathematics education study program at Riau Islands University with the context of typical Riau Islands snacks, namely three types of typical snacks used, namely luti gendang cake, deram-deram cake and cakar ayam cake. A total of 15 students were divided into 5 groups, representing diverse ability levels, namely high, medium and low. Data were collected through documentation and interviews, with data analysis techniques following the Miles and Huberman approach (as described by Lestary & Sani , 2018). The analysis process involved data reduction, data presentation, and conclusion drawing as the main steps.

RESULTS AND DISCUSSION

Preparing For The Experiment

In the preparing for the experiment stage, HLT development for each learning activity is the most important part in designing student learning activities. The design is inseparable from the learning trajectory which contains the learning plan of the material to be taught. HTL design in table 1.

Table 1. *Hypothetical Learning Trajectory* Design

Akt	Learning Objectives	Learning Activities	Predicted Answer	Anticipated Answer
1	Recalling SPLDV material	Given a contextual problem related to the purchase price of luti gendang cakes. Given the price of all cakes purchased, then students are asked to determine the price of 1 pcs of luti gendang.	• Students use an approximate approach in determining the answer. • Students find solutions to problems by outlining possible solutions without modeling. • Students create a mathematical model of the given	Provide some Question: • What if the number of cakes purchased is larger? Isn't it difficult to just rely on estimated prices? • Previously, have we studied Linear Equations of

			problem and then solve the problem.	One Variable (PLSV)? • How can we simplify the mathematical representation of a cake into a simpler form?
2	Students recall the SPLDV material	Given a contextual problem related to the purchase price of luti gendang and deram-deram cakes. Given the price of all cakes purchased, then students are asked to determine the price of 1 pcs of deram-deram cake.	• Students use estimation in determining the answer. • Students solve problems by listing possible solutions without modeling. • Students create a mathematical model of the given problem, then solve the problem using the substitution method.	Ask a few questions: • What if you buy more cakes? Isn't it hard to just guess the price? • Have you ever studied the System of Linear Equations of Two Variables (SPLDV) before? • How can you simplify both types of cakes into simpler forms in a mathematical context?
3	Students are able to make mathematical models of problems related to SPLTV	Given a contextual problem related to the total price of 3 types of cakes purchased in different quantities, students are asked to determine the price of 1 box of cakes.	• Students solve the problem in a simple way without making a mathematical model of the given problem. • Students record possible answers by assuming the price of all cakes is the same. • Students create a mathematical	Ask a few questions: • Does the question say that the price of each type of cake is the same? • What if more cakes are purchased? Is it not difficult to just speculate on the price?

	model of the given problem.	• How can we model the problem in a simple form?

Design Experiment and Restrospective Analysis

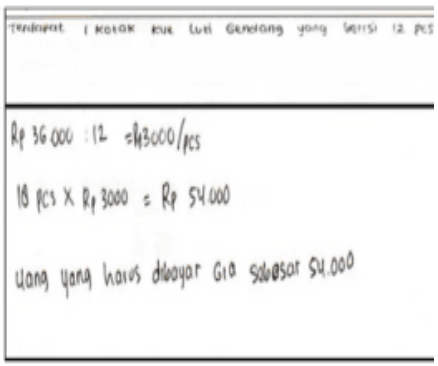
In the first meeting, students are expected to recall the material that has been studied before, namely about linear equations of one variable and linear equations of two variables. Given the problem in Figure 2.

From the picture, write down what information you get! If Bilqis buys one box of luti gendang for IDR. 36,000 and Gia wants to buy 18 pcs of luti gendang, how much money should Gia pay?

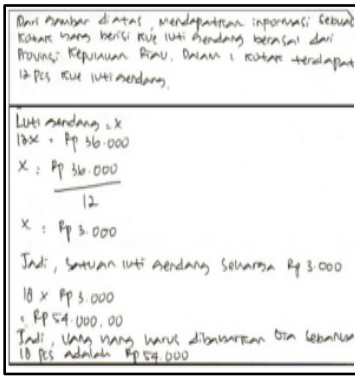
Figure 2. Luti Gendang cake

From the problems given, the following are the students' answers.

Students work on activity 1



Answer type 1



Answer type 2

Figure 3. Activity 1

In Figure 3, students used the division operation to determine the price of 1 luti drum. This type 1 answer is not perfect because students have not replaced the object in question by using a variable according to the concept of a one-variable linear equation. There were two groups of students with similar answers. The researcher then guided the students to solve the problem by applying the concept of one variable linear equation.

Then there were three groups of students who solved the problem using the concept of linear equation of one variable. In type 2 answers, students have solved the problem using

the concept of linear equations of one variable, students have understood modeling mathematical problems using linear equations of one variable.

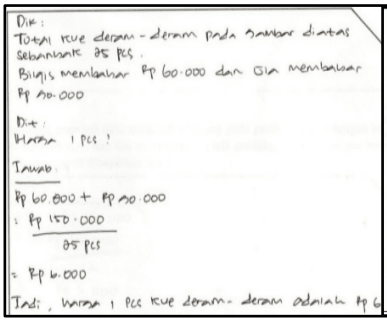
In the second activity students are expected to recall the material on linear equations of two variables. Given the contextual problem as follows.

If Bilqis and Gia also buy deram-deram cakes, so Bilqis has to pay Rp. 60,000 and Gia has to pay IDR. 90,000. How much does 1 pcs of deram-deram cake cost?

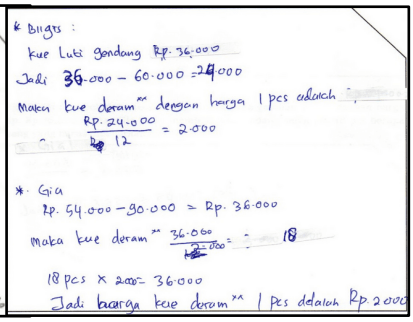
Figure 4. Deram-deram Cake

From the problems given, the following are the students' answers.

Students work on activity 2



Answer type 1



Answer type 2

Figure 5. Activity 2

In Figure 5, type 1 students answered by adding up all the deram-deram cakes and the money to be paid. They used the principle of division to find the price of 1 deram-deram cake. However, this answer is incorrect because students ignore the information from the previous step in the problem and have not applied the concept of linear equations of two variables.

Answer type 2 students calculated the price of 1 piece of kue deram-deram by subtracting the total purchase of Bilqis and Gia's luti gendang from the total payment after adding the purchase of kue deram-deram. They used the principle of division to find the price of 1 piece of kue deram-deram. However, this answer was still incomplete because the students had not applied the concept of two-variable linear equations and there were errors in writing the subtraction. The researcher asked follow-up questions to explore further understanding.

L : From the answer written, is it correct that IDR.36,000 - IDR.60,000 = IDR.24,000. as well as for Gia's calculation?

S : No ma'am, it's just a writing mistake because I answered in a hurry, what should be written is the total Bilqis has to pay IDR.60,000 minus the purchase price of luti gendang IDR.36,000, - so the result is IDR. 24,000 as the total price for the purchase of Deram-deram cake

L : How to calculate the price of 1 pcs of deram-deram cake?

S : IDR. 24,000 divided by the number of 1 box of deram-deram cakes, from the picture that is seen that is already in the box there are 12 pcs of deram cakes, so

IDR.24,000 divided by 12 the result is IDR.2000, - for the price of 1 pcs of deram-deram cakes.

L : Previously, I had learned SPLDV at school.

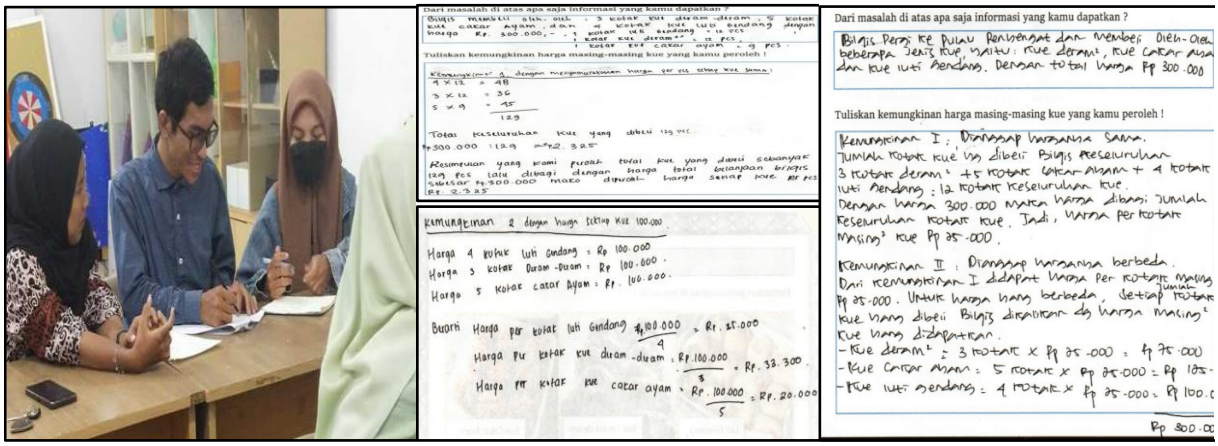
S : Already ma'am, the ones that memorize with the x and y variables are yes ma'am, then to calculate it can use substitution and elimination

L : Yes, please continue the solution

In the third activity students are expected to be able to model SPLTV, students are given the following problem.

Bilqis traveled to Penyengat Island and bought 3 boxes of Deram-deram cookies, 5 boxes of Cakar Ayam cake and 4 boxes of Luti Gendang for IDR.300,000. What is the possible price for 1 box of Deram-deram cake, 1 box of Cakar Ayam cake and 1 box of Luti Gendang cake?

Here are the students' answers to the problems given.



Students work on activity 3 Answer Type 1 Answer Type 2

Figure 6. Activity 3

In Figure 6, student type 1 answers by calculating the total number of cakes purchased, so that the price of one cake is IDR.2,325. And then the student calculated the price of each box by dividing the total price of cake purchases by three types of cakes. So that the price per box of cake is obtained. The answer is still not correct because the total purchase price of the cake is less than IDR.300,000, -. then the answer type 2 students answer by assuming the price of each box of cake is the same, students divide the total purchase price of IDR.300,000 by the total number of boxes of cake, namely 12 boxes. So that the price of 1 box of cake is IDR.25,000, -. the student's answer is still not correct because the question does not mention the price of each box of cake is the same. For this student's answer, the researcher gave a follow-up question:

L : Is the question stated that the price of each cake is the same?
S : no ma'am
L : If not then there are other possible prices for each box of cake?
S : Yes ma'am
L : How do you calculate the other possibilities?
S : That means considering the total purchase price of the cake and the number of cakes. purchased for each type (It takes a long time for students to answer this).
L : Right, please continue to calculate other possibilities.

Furthermore, the answers of type 3 students are shown in Figure 7.

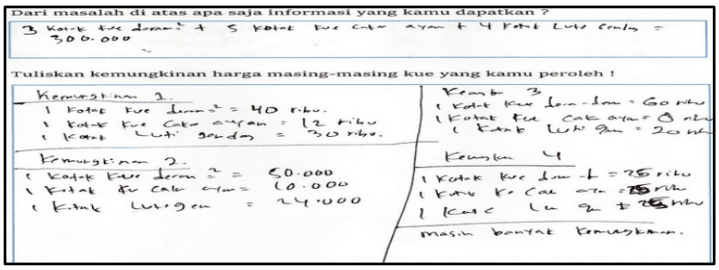
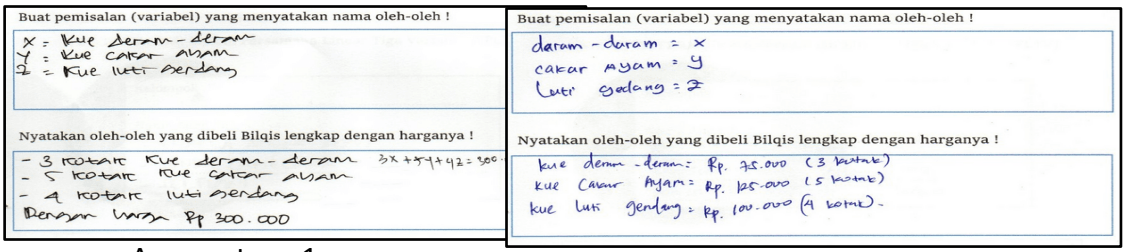


Figure 7: Type 3 answer

In Figure 7, students answered by. Writing down the various possible prices that satisfy. There are four possible cake prices written down and the student also wrote down many other possible prices. In the next activity, students were asked to create a variable that states the name of the souvenir and continued by stating the souvenir Bilqis bought after creating a variable complete with the price. The following is the answer of type 1 students in Figure 8.



Answer type 1 Answer type 2

Figure 8. Activity 3

In Figure 8, type 1 answers students have made variable memorization, namely, kue deram-deram x, kue cakar ayam y and kue luti gendang z. But when stating the souvenirs purchased complete with the price, the students did not write in the form of a linear equation of three variables. In type 2 answers, students have made a variable memorization, namely, kue deram-deram x, kue cakar ayam y and kue luti gendang z. And when stating the souvenirs purchased complete with the price, the students wrote it in the form of a linear equation of three variables. And when stating the souvenirs purchased complete with the price, the student has written in the form of a three-variable linear equation. But it is still incomplete to write the equation to determine the price of 1 city for the three types of cakes. For this answer, the researcher asked a follow-up question.

L : From the equation obtained, we will discuss what material?
S : Lineart equation of three variables bu
L : right, try from the existing equation can you make it general to calculate the price of each box of cookies?
S : Does that mean finding the number of x, y and z?
L : Yes.
S : x equals IDR.300.000,- minus 5y minus 4z divided by 3, and the same goes for y and z.

The results of experiments in learning showed that the *Hypothetical Learning Trajectory* (HLT) designed in accordance with students' learning trajectory was effective in facilitating understanding of the system of linear equations of three variables. Students succeeded in linking their learning experience in the material of Linear Equation One Variable and Linear Equation Two Variables with the new material they would learn, namely SPLTV. This approach allows students to feel the connection between the mathematical concepts they are learning, opening up opportunities for a deeper and more integrated understanding.

In solving contextual problems, students use various methods, ranging from simple ones to connecting them with relevant theories they have learned before. Research (Lusiana, 2017) shows that the variety of solution methods by students for contextual-based linear equation system problems can overcome problems with different approaches. Nenohai, et al (2024) confirmed that students with good mathematical performance showed problem understanding, planning, implementation, and accurate solutions according to the context. Research by Ratuanik et al. (2021) showed that the Indonesian Realistic Mathematics Learning (PMRI) approach improved students' understanding, while Sabil & Winarni (2013) and Sari & Noviartati (2017) highlighted the improvement of students' abilities through the application of PMRI.

Realistic Mathematics Education (RME)-based learning trajectories have also been successfully applied in various mathematics materials, such as geometry, anti-derivatives, fractions, and enumeration methods. In this study, PMRI-based learning trajectories were piloted on the topic of SPLTV, which has a strong relevance to the real world. This approach brings students into problem situations often encountered in everyday life, allowing them to relate mathematics to their surroundings and personal experiences. Through PMRI, students can embrace mathematics as an activity that is integrated with the surrounding reality.

CONCLUSION

The results of this study show that in learning SPLTV, learning design can be used using local contexts such as typical snacks or other things that are easily found in students' daily activities. Thus, students can understand mathematical concepts easily, fun, close to students' daily activities and affordable by students' imagination. This also makes it easier for students to be able to solve problems encountered in students' daily lives. This shows that learning SPLTV with the PMRI approach does not only focus on theoretical aspects, but also encourages

students' ability to relate and apply mathematical concepts in the context of everyday life. Thus, this approach improves students' understanding in solving mathematical problems.

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