Jurnal Math Educator Nusantara
Wahana publikasi karya tulis ilmiah di bidang pendidikan matematika
p-issn: 2459-9735 e-issn: 2580-9210
http://ojs.unpkediri.ac.id/index.php/matematika

# Pre-service Mathematics Teachers' Understanding of Vector

**Kimura Patar Tamba[1] ***

[1]Program Studi Pendidikan Matematika, Universitas Pelita Harapan. MH Thamrin Boulevard 1100, Klp. Dua, Kec. Klp. Dua, Tangerang, Indonesia.
E-mail:[1] kimura.tamba@uph.edu

Article received : *September 24, 2022*,
article revised : *April 10, 2023*,
article Accepted: *April 13, 2023*
* Corresponding author

**Abstract**: Vector is an important concept in mathematics. Pre-service mathematics teachers explore vector concepts in several courses. This concept is also related to various concepts in mathematics. The purpose of this study is to describe the pre-service mathematics teachers' understanding of vectors. This research is non-experimental research with a descriptive design. 67 pre-service mathematics teachers are participants in this study. Data were collected by survey method using the Test of Understanding of Vector instrument. Data analysis was carried out quantitatively using descriptive and inferential statistics. The descriptive statistics used are the average, maximum, minimum, and standard deviation. The inferential statistics used are t-test statistics. The results showed that in general, the pre-service mathematics teachers' understanding of vectors was in the medium category. The results also show that there is no difference in understanding of vectors between male and female pre-service mathematics teachers. However, in the two concepts, namely "unit vector" and "scalar multiplication" there are significant differences between males and females.

**Keywords**: Test of Understanding of Vector; Vector; Pre-Service Mathematics Teachers

**Pemahaman Materi Vektor Guru Matematika Prajabatan**

**Abstrak:** Vektor merupakan konsep penting dalam matematika. Calon guru matematika mendalami konsep vektor dalam beberapa mata kuliah. Konsep ini juga berhubungan dengan berbagai konsep lain dalam matematika. Tujuan penelitian ini adalah untuk mendeskripsikan pemahaman calon guru matematika mengenai vektor. Penelitian adalah penelitian non-experimental dengan desain deskriptif. Terdapat 67 calon guru yang menjadi partisipan dalam penelitian ini. Data dikumpulkan dengan metode survei dengan menggunakan instrumen Test of Understanding of Vector. Analisis data dilakukan secara kuantitatif dengan memakai statistik deskriptif dan inferensial. Statistik deskriptif yang digunakan adalah rata-rata, maksimum, minimum dan standar deviasi. Statistik inferensial yang digunakan adalah statistik uji-t. Hasil penelitian menunjukan secara umum pemahaman calon guru matematika mengenai vektor berada pada kategori sedang. Hasil penelitian juga menunjukkan tidak terdapat perbedaan pemahaman mengenai vektor antara calon guru matematika laki-laki dan perempuan. Meskipun begitu pada dua konsep yaitu "unit vector" dan "scalar multiplication" terdapat perbedaan signifikan antara laki-laki dan perempuan.

**Kata Kunci**: Test of Understanding of Vector; Vector; Pre-Service Mathematics Teachers

## INTRODUCTION

Vector is an important concept in mathematics. In mathematics for teacher education at universities, vectors are taught in linear algebra, calculus, multiple-variable calculus, and analytical geometry (Stewart, 2012; Weir & Hass, 2014). In the field of Biology, the concept of vectors is also important (Stewart & Day, 2015). In addition, vectors are very

**CITATION FORMATS:** Tamba, K. P. (2023). Pre-service mathematics teachers' understanding of vector. *Jurnal Math Educator Nusantara: Wahana Publikasi Karya Tulis Ilmiah Di Bidang Pendidikan Matematika*, *9*(1). 15-24 https://doi.org/10.29407/jmen.v9i1.18708

important concepts in physics, even many quantities in physics are expressed in vectors (Oktavianty, 2021; Taqwa & Rahim, 2022).

Various studies have shown that understanding vectors is very important (Barniol & Zavala, 2013, 2014c, 2014b; Deprez et al., 2019; Oktavianty, 2021). Even the development of instruments that measure understanding of vectors at the university level was carried out (Barniol & Zavala, 2013, 2014b; Pratama et al., 2018; Rakkapao et al., 2016; Susac et al., 2018). The result is an instrument called the Test of Understanding of Vector (TUV). The use of TUV has provided a comprehensive overview and analysis of the understanding of vectors. These results are important in developing the learning process about vectors in university.

However, various studies on vector understanding are still focused on undergraduate students majoring in physics (both pre-service teachers and not) ((Barniol & Zavala, 2012, 2014a; Deprez et al., 2019; Jewaru et al., 2021; Oktavianty, 2021; Saraçoglu & Kol, 2018; Taqwa & Rahim, 2022; Zavala & Barniol, 2013). The use of TUV to measure understanding of vectors in the context of mathematics education has not been carried out. It is very important to know and analyze the pre-service mathematics teachers' understanding of vectors. Therefore, it is very important to conduct research using TUV in analyzing the pre-service mathematics teachers' understanding of vectors. The TUV development framework contains concepts regarding direction, magnitude, component, unit vector, vector representation, addition, subtraction, scalar multiplication, dot product, and vector product (Barniol & Zavala, 2013, 2014b). This framework fits perfectly with the scope and sequence of vector learning for mathematics and mathematics education at universities (Stewart, 2012; Weir & Hass, 2014). In other words, TUV is suitable for measuring pre-service mathematics teachers' understanding of vectors.

The purpose of this study is to describe the pre-service mathematics teachers' understanding of the vector concept. Accordingly, this study will also analyze the differences in the understanding of pre-service mathematics teachers regarding vector concepts based on gender.

## METHOD

This research is a non-experimental quantitative research with a descriptive design. This design was chosen because this study aims to describe a phenomenon as it is without intervention (Cohen et al., 2018; Gall et al., 2003). In this case, the phenomenon that will be described is the conceptual ability of pre-service mathematics teachers regarding vectors. The participants in the study were 67 pre-service mathematics teachers at a university in Tangerang, Indonesia. This participant has completed courses in linear algebra, differential calculus, integral calculus and multivariable calculus, each of which involves material about vectors. The selection of participants was carried out purposively, namely prospective teachers who had completed courses related to vectors.

The instrument used to measure understanding of vectors is the Test of Understanding of Vectors (TUV). This TUV was developed by Barniol & Zavala (2013). TUV consists of 20 items that measure 10 concepts regarding vectors, namely direction (5, 17), magnitude (20),

component (4, 9, 14), unit vector (2), vector representation (10), Addition (1, 7, 16), Subtraction (13, 19), Scalar multiplication (11), Dot product (3, 6, 8), Vector product (12, 15, 18) (Barniol & Zavala, 2013, 2014b). Of the 20 TUV items, there are 11 items regarding vector concepts in graphic form (items 1-5, 9-13, 19), 7 items regarding calculation of vector concepts (6, 8, 14, 15, 17, 18, 20), and 2 items that cover graphical and calculations aspects (7, 16). In detail, the TUV instrument can be seen in Table 1.

Table1. *Framework of Test of Understanding of Vector (TUV)*

| No. | Concept | Item | Description |
|---|---|---|---|
| 1. | Direction | 5 | Choosing a vector with the same direction from among several in a graph |
| | | 17 | Calculation of direction of a vector written in unit-vector notation |
| 2. | Magnitude | 20 | Calculation of magnitude of a vector written in unit-vector notation |
| 3. | Component | 4 | Graphic representation of y component of a vector |
| | | 9 | Graphic representation of x component of a vector |
| | | 14 | Calculation of x component of a vector (angle measured from y axis) |
| 4. | Unit vector | 2 | Graphic representation of a unit vector |
| 5. | Vector representation | 10 | Graphic representation of a vector written in unit-vector notation |
| 6. | Addition | 1 | Graphical addition of vectors in 2D |
| | | 7 | Comparing the vector sum's magnitude of two same-magnitude vectors at 90° with the magnitude of the vectors. |
| | | 16 | Comparing the vector sum's magnitude of two same-magnitude vectors at 143.13° with the magnitude of the vectors. |
| 7. | Subtraction | 19 | Graphical subtraction of vectors in 1D |
| | | 13 | Graphical subtraction of vectors in 2D |
| 8. | Scalar multiplication | 11 | Graphic representation of a vector multiplied by a negative scalar |
| 9. | Dot product | 3 | Geometric interpretation of dot product as a projection |
| | | 6 | Calculation of dot product using the equation $AB\cos\theta$ |
| | | 8 | Calculation of dot product of vectors written in unit-vector notation |
| 10. | Vector product | 12 | Geometric interpretation of cross product as a perpendicular vector |
| | | 15 | Calculation of cross product of vectors written in unit-vector notation |
| | | 18 | Calculation of a cross product magnitude using the equation $AB\sin\theta$ |

This TUV is in the form of multiple choice with a single correct answer and is given a score of 1, and the wrong answer is given a score of 0. Details of the TUV items can be seen in the paper of Barniol & Zavala (2013). Various studies have shown that TUV has acceptable validation and reliability (Bani-salameh et al., 2020; Barniol & Zavala, 2013, 2014b; Oktavianty, 2021; Susac et al., 2018; Whelan et al., 2021).

However, the researchers still conducted validation and reliability tests on TUV in this research context. Construct validity is carried out by providing the TUV framework to experts for review. The experts who tested the construct validity were Calculus and Linear Algebra lecturers and research methods (measurement) lecturers. Experts were asked to assess the theoretical framework, definition, construction of each item and the clarity of each item whether it was valid or invalid. The expert also provides comments on each item or proposes additional items. The results of construct validity show that all experts agree that each test item on the TUV is in accordance with the concepts and indicators from the TUV, and is in accordance with the curriculum regarding vectors for pre-service mathematics teachers. The point-biserial coefficient is a measure of the consistency of a single item with the whole test. Validation test also using a point-biserial coefficient. The results of the validation test show that the $r_{pbs}$ value is in the range of 0.2 to 0.7, in addition to item 18 which has an $r_{pbs}$ value of -0.121. Therefore, item 18 was excluded from the TUV. This questionnaire was given to participants using Moodle. The time given to complete it is two hours. The reliability test was carried out with Cronbach's Alpha. Cronbach's Alpha value is 0,747 ($> 0{,}6$) indicating that TUV has adequate reliability.

TUV data were analyzed using descriptive and inferential statistics. Descriptive statistical values (mean, std.dev, max, min) of TUV will be calculated in total items and items based on concepts. In addition, the mean value of TUV results will be categorized into high, medium, and low. This categorization uses hypothetical statistical techniques (Sugiyono, 2012) as shown in Table 2.

Table 2. *Category of Mean*

| No. | Category | Mean |
|---|---|---|
| 1. | High | $\bar{X} > 0.667$ |
| 2. | Medium | $0.33 < \bar{X} \leq 0.667$ |
| 3. | Low | $\bar{X} \leq 0.333$ |

The inferential statistic used is the independent samples t-test statistic. The aim is to get an idea of whether there are differences in abilities based on gender. Before conducting the independent samples t-test statistic, an assumption test is carried out, namely the normality test and homogeneity test. The normality test was carried out using the Shapiro-Wilk test. The results of the data normality test showed that the data came from a normal distribution, where the value of $p > 0{,}05$ (table 3). Homogeneity test was carried out with Levene's test. The results of the homogeneity test showed that there was no difference in

the variance in the ability to understand concepts between men and women, where $p > 0{,}05$ (see table 3). The analysis was carried out using SPSS 20.0 software.

Table 3. *Assumption test*

| No. | Assumption | Value | *p* |
|---|---|---|---|
| 1. | Normality | Male: $W = 0{,}943$ | 0,188 |
| | | Female: $W = 0{,}965$ | 0,204 |
| 2. | Homogeneity | $F = 0{,}209$ | 0,649 |

**RESULT AND DISCUSSION**

The research results will be presented in two major parts. First, the results of a descriptive analysis regarding the level of understanding of pre-service mathematics teachers regarding vectors. Second, the results of the comparative test of pre-service mathematics teachers' understanding of vectors by gender. The results of the research on the description of the understanding of mathematics teacher candidates regarding vectors are shown in Table 4. The description of understanding in Table 4 is the result of descriptive statistical analysis of the TUV data given to prospective mathematics teachers. The mean value in this analysis can be seen as the proportion of the number of TUV questions that can be completed by pre-service mathematics teachers. The higher the mean value ($0 \leq mean \leq 1$), the higher the number of TUV questions that the pre-service mathematics teachers can solve. In total, the average TUV score is 0.497 ($< 0.5$). This means that pre-service mathematics teachers are only able to complete about 50% of the total of TUV questions. If viewed based on the concept domain, the TUV questions regarding scalar multiplication were the most difficult for pre-service mathematics teachers to solve (mean = 0.224).

Tabel 4. *Description of Pre-service Mathematics Teachers' Level of Understanding on Vector*

| No. | Concept | Mean | Std.Dev | Max | Min | Category |
|---|---|---|---|---|---|---|
| 1. | Total | 0.497 | 0.212 | 1.000 | 0.158 | Medium |
| 2. | Direction (TUV-01) | 0.649 | 0.369 | 1.000 | 0.000 | Medium |
| 3. | Magnitude (TUV-02) | 0.448 | 0.501 | 1.000 | 0.000 | Medium |
| 4. | Component (TUV-03) | 0.423 | 0.346 | 1.000 | 0.000 | Medium |
| 5. | Unit vector (TUV-04) | 0.746 | 0.438 | 1.000 | 0.000 | High |
| 6. | Vector representation (TUV-05) | 0.478 | 0.503 | 1.000 | 0.000 | Medium |
| 7. | Addition (TUV-06) | 0.483 | 0.274 | 1.000 | 0.000 | Medium |
| 8. | Subtraction (TUV-07) | 0.545 | 0.387 | 1.000 | 0.000 | Medium |
| 9. | Scalar multiplication (TUV-08) | 0.224 | 0.420 | 1.000 | 0.000 | Low |
| 10. | Dot product (TUV-09) | 0.463 | 0.312 | 1.000 | 0.000 | Medium |
| 11. | Vector product (TUV-10) | 0.530 | 0.398 | 1.000 | 0.000 | Medium |

The mean value also indicates the level of understanding. The higher the mean value ($0 \leq mean \leq 1$), the better the pre-service mathematics teachers' understanding of vectors

is. This can be seen in the categorization of the level of understanding of pre-service mathematics teachers in Table 4. The level of understanding of pre-service mathematics teachers of vectors is in the average category. In terms of the vector concept domain, only the concept of "unit vector" is in the high category. This means that pre-service mathematics teachers have a high understanding of the concept of unit vectors. The test questions regarding the unit vector concept are expressed in a graphic representation. Therefore, in detail, these results show that pre-service mathematics teachers have a high understanding of the graphic representation of a unit vector. Meanwhile, the results of this TUV show that pre-service mathematics teachers still have difficulties with the concept of "scalar multiplication". This can be seen from the category the level of understanding of "scalar multiplication" is in a low category. When viewed from the form of the test questions, this result also shows the low understanding of pre-service mathematics teachers regarding scalar multiplication which is expressed in the form of graphic representation.

Tabel 5. *Percentage by Category Level of Understanding*

| No. | Concept | High | Medium | Low |
|---|---|---|---|---|
| 1. | Total | 28% | 46% | 25% |
| 2. | Direction (TUV-01) | 16% | 37% | 46% |
| 3. | Magnitude (TUV-02) | 55% | 0% | 45% |
| 4. | Component (TUV-03) | 27% | 57% | 16% |
| 5. | Unit vector (TUV-04) | 25% | 0% | 75% |
| 6. | Vector representation (TUV-05) | 52% | 0% | 48% |
| 7. | Addition (TUV-06) | 12% | 79% | 9% |
| 8. | Subtraction (TUV-07) | 25% | 40% | 34% |
| 9. | Scalar multiplication (TUV-08) | 78% | 0% | 22% |
| 10. | Dot product (TUV-09) | 21% | 69% | 10% |
| 11. | Vector product (TUV-10) | 28% | 37% | 34% |

The results of the descriptive statistical analysis above are in line with the percentage of participants who have an understanding of vectors in the high, medium, or low categories (see Table 5). When viewed per participant, in total most (46%) have an understanding of vector concepts in the medium category. The same thing is also obtained when viewed based on the TUV concept domain. Only in the concept of "magnitude", "Vector representation" and "scalar multiplication" where the percentage of the number of pre-service mathematics that has an understanding in the high category is greater. In other concept domains, most (more percentages) of pre-service mathematics have an understanding of the medium category.

Tabel 6. *Value of T-test*

| Domain | t | df | p |
|---|---|---|---|
| Total | -1.383 | 65 | 0.171 |
| Direction (TUV-01) | -1.448 | 65 | 0.152 |
| Magnitude (TUV-02) | 0.635 | 65 | 0.528 |
| Component (TUV-03) | 0.623 | 65 | 0.535 |
| Unit vector (TUV-04) | 1.827 | 65 | 0.072* |
| Vector representation (TUV-05) | -1.782 | 65 | 0.079 |
| Addition (TUV-06) | -1.485 | 65 | 0.142 |
| Subtraction (TUV-07) | -2.443 | 65 | 0.017 |
| Scalar multiplication (TUV-08) | -1.452 | 65 | 0.151* |
| Dot product (TUV-09) | -0.900 | 65 | 0.371 |
| Vector product (TUV-10) | -1.101 | 65 | 0.275 |

*signifikan pada $\alpha = 0.05$

The results of the analysis of the average test with t-test statistics are shown in Table 6. The results of this analysis show that in total there is no significant difference between males and females in understanding the vector concept. In terms of the concept domain, only "substraction" have significant differences between males and females. While in other domains, there is no significant difference between male and female mathematics teacher candidates in understanding vector concepts.

Based on the results of the research above, the findings of this study are that the level of understanding of pre-service mathematics teachers regarding vectors is in the medium category. The medium category means that pre-service mathematics teachers are only able to complete about 50% of the total number of TUV tests. The results of Barniol & Zavala (2014) research show the same thing. Barniol & Zavala (2014)) show that the ability of undergraduate students (pre-service teachers in this category) is able to complete 68% of the number of TUV test questions which is also in the medium category. Other studies also show the same thing, that undergraduate understanding (who studies vector topics) regarding vectors is still in the medium category (Barniol & Zavala, 2012; Saraçoglu & Kol, 2018).

Based on the domain, pre-service mathematics teachers have a low understanding of scalar multiplication. While in other domains the understanding of pre-service mathematics teachers is in the medium or high category. This result is different from previous research which shows that undergraduate students have difficulty in unit vector domains, cross and dot product, subtraction and direction (Rakkapao et al., 2016), cross product, subtraction of vectors, dot product, and the direction of a vector (Susac et al., 2018), vector addition (Bollen et al., 2017). This result is also different from previous research which shows that pre-service mathematics teachers who have studied vectors still have difficulty in the topic of vector subtraction that does not display vectors initially, and determining the direction of vectors (Saputri et al., 2019). However, the results of this study are also in line with other studies which show that students who have studied vectors also have difficulty in the scalar

multiplication domain (Barniol & Zavala, 2012; Saraçoglu & Kol, 2018).. The differences in the results of this study indicate that the level of difficulty of the participants regarding the concept is different.

The low ability in scalar multiplication may occur because of the failure to make connections between concepts and formal, as also found in previous studies (Zavala & Barniol, 2013). In TUV, the problem of scalar multiplication is expressed in an image representation, not in a formal form. Therefore, when pre-service mathematics teachers do not understand the concept, it will be difficult for them to relate the representation to the formal form

Another finding from this study is that in total there is no difference in the ability to understand vectors based on gender. However, there are differences in the understanding of vectors between females and males in the concept domain of "substraction." The results of this study are different from previous studies (Saraçoglu & Kol, 2018). The results of research by Saraçoglu & Kol (2018) show that there are differences in understanding of vectors between females and males in certain domains.

The results of this study should be understood in terms of several limitations. First, the number of study participants limits the generalizability. Second, the understanding of the claim that the cause of the low understanding of pre-service mathematics teachers regarding the concept of "scalar multiplication" is due to the failure to make connections between concepts and formal needs to be clarified through the following research.

## CONCLUSION

Based on the findings and discussion, two things can be concluded from this research. First, in general, the pre-service mathematics teachers' understanding of vectors is in the medium category. Second, in general, there is no difference in the understanding of vectors between men and women. However, in the concept domain of "substraction" there are significant differences between males and females.

The implication of the results of this study is the need to review and develop courses and the lecture process on the teacher's campus related to vectors. For example in linear algebra, analytic geometry, calculus, and multivariable calculus. In addition, the learning process regarding vectors must be carried out comprehensively both in terms of representation and context.

## REFERENCE

Bani-salameh, H. N., Alkofahi, K. A., & Al-khateeb, H. M. (2020). ScienceDirect Investigating the Knowledge of Vector Concepts of First Year Medical Students. *Health Professions Education*, *6*(3), 428–433. https://doi.org/10.1016/j.hpe.2020.05.001

Barniol, P., & Zavala, G. (2012). Students' difficulties with unit vectors and scalar multiplication of a vector. *AIP Conference Proceedings*, *1413*(February), 115–118. https://doi.org/10.1063/1.3680007

Barniol, P., & Zavala, G. (2013). *Testing Students ' Understanding of Vector Concepts*. 65–68. https://doi.org/10.1119/perc.2013.pr.004

Barniol, P., & Zavala, G. (2014a). Students ' difficulties in problems that involve unit-vector notation. *Latin American Journal of Physics Education*, *8*(4), 1–10.

Barniol, P., & Zavala, G. (2014b). *Test of understanding of vectors : A reliable multiple-choice vector concept test*. *010121*, 1–14. https://doi.org/10.1103/PhysRevSTPER.10.010121

Barniol, P., & Zavala, G. (2014c). Test of understanding of vectors: A reliable multiple-choice vector concept test. *Physical Review Special Topics - Physics Education Research*, *10*(1), 1–14. https://doi.org/10.1103/PhysRevSTPER.10.010121

Bollen, L., Van Kampen, P., Baily, C., Kelly, M., & De Cock, M. (2017). Student difficulties regarding symbolic and graphical representations of vector fields. *Physical Review Physics Education Research*, *13*(2). https://doi.org/10.1103/PhysRevPhysEducRes.13.020109

Cohen, L., Manion, L., & Morrison, K. (2018). *Research Methods in Education*. Routledge.

Deprez, T., Gijsen, S. E., Deprez, J., & De Cock, M. (2019). Investigating student understanding of cross products in a mathematical and two electromagnetism contexts. *Physical Review Physics Education Research*, *15*(2), 20132. https://doi.org/10.1103/PhysRevPhysEducRes.15.020132

Gall, M. D., Gall, J. P., & Borg, W. R. (2003). *Educational Research_ An Introduction (7th Edition)*. Pearson.

Jewaru, A. A. L., Umrotul, Kusairi, S., & Pramono, N. A. (2021). Senior high school students understanding of vector concepts in mathematical and physical representations. *AIP Conference Proceedings*, *050024*(March), 1–9. https://doi.org/https://doi.org/10.1063/5.0043433

Oktavianty, B. E. (2021). *Analysis of Pre-Service Physics Teachers ' Understanding of Vectors and Forces*.

Pratama, A. C., Supahar, Warsono, & Jumadi. (2018). The development physics essay test to measure vector and mathematics representation ability in senior high school. *Journal of Physics: Conference Series*, *1097*(1). https://doi.org/10.1088/1742-6596/1097/1/012013

Rakkapao, S., Prasitpong, S., & Arayathanitkul, K. (2016). *Analysis test of understanding of vectors with the three-parameter logistic model of item response theory and item response curves technique*. *020135*, 1–10. https://doi.org/10.1103/PhysRevPhysEducRes.12.020135

Saputri, D. E., Shodiqin, M. I., & Kurniawan, B. R. (2019). Pemahaman Konsep Vektor Mahasiswa Calon Guru. *Seminar Nasional Pendidikan Fisika Dan Pembelajarannya*, 41–47.

Saraçoglu, H., & Kol, Ö. (2018). *Evaluation of teacher candidates ' knowledge about vectors*. *19*(2).

Stewart, J. (2012). *Calculus: Early Transcendentals* (7E ed.). Brooks/Cole.

Stewart, J., & Day, T. (2015). *Biocalculus: Calculus, Probability, and Statistics for the Life Sciences*.

Sugiyono. (2012). Metode Penelitian Kuantitatif, Kualitatif dan R&D. In *Alfabeta*. Alfabeta.

Susac, A., Planinic, M., Klemencic, D., & Sipus, Z. M. (2018). Using the Rasch model to analyze the test of understanding of vectors. *Physical Review Physics Education Research*, *14*(2), 23101. https://doi.org/10.1103/PhysRevPhysEducRes.14.023101

Taqwa, M. R. A., & Rahim, H. F. (2022). Students' conceptual understanding on vector topic in visual and mathematical representation: A comparative study. *Journal of Physics: Conference Series*, *2309*(1). https://doi.org/10.1088/1742-6596/2309/1/012060

Weir, M. D., & Hass, J. (2014). *Thomas' Calculus*. Pearson.

Whelan, E. T., Ray, T. P., Kattnig, D. R., Nielsen, C., & Abyaneh, M. Y. (2021). *Classical test theory and Rasch analysis of test of understanding of vectors ( TUV ) Classical test theory and Rasch analysis of test of understanding of vectors ( TUV )*. https://doi.org/10.1088/1742-6596/1719/1/012089

Zavala, G., & Barniol, P. (2013). Students' understanding of dot product as a projection in no-context, work and electric flux problems. *AIP Conference Proceedings*, *1513*(August 2014), 438–441. https://doi.org/10.1063/1.4789746