Three zone learning concepts to improve mathematical proof of probability theory

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Abstract: This study aims to comprehensively analyze the improvement of students' mathematical development ability through the concept of three zone learning, with the type of research is quantitative and quasi experiment design. Examples of research as many as two classes amounted to 82 pre service teacher. Data analysis by calculating gain normalization. Results: based on the Post Hoc test, the level of the category that has a positive score of 0.012 with the upper category and the middle category means that the average score of students from the upper category is better than the middle category, then also for the level that has a positive score of 0.028 with the upper category and the lower category which means that the average score of students from the upper category is better than the lower category. As for the positive value of 0.016 with the middle category and the lower category means the average of the middle category is better. From the results showed that for levels with upper, middle and lower categories, the effect is greater in improving students.

Keywords: Three Zone Learning; Mathematical Proof; Probability Theory

INTRODUCTION

Pre-service teacher need to master mathematical skills in the process of learning in college, one of which is the ability of mathematical proof. Arnawa, Yerizon & Enita (2019) suggests that mathematical proof is a sequence of logical representation and statement which explain why a given proposition is true. Studying mathematical proofs can plays a central role in developing, establishing and communicating mathematical and can help students understand concepts (Rocha, 2019). Mathematical proof ability is part of Mathematical Reasoning Ability (Brodie, 2010; Hendriana, Rohaeti & Sumarmo, 2017). In fact, mathematical proof is a very
difficult thing, students found difficulties in solving problems involving mathematical proof (Agustiyaningrum et al., 2020). Mathematical proof is one aspect that must be considered learning mathematics in school and one of the obstacles student experience is difficulty in constructing evidence (Hamimi & Sari, 2018). Hamid (2016) suggests that students still have difficulty in mathematical proof and still below the expected level. The ability of mathematical proof, students are still low, especially in formulating how to solve a mathematical proof, the obstacles for students in studying mathematics include the difficulty of students in doing mathematical proofs, so it can be said that the ability of mathematical proof is low (Mubarak, Pujiastuti & Suparsih, 2018). Likewise, according to (Nurrahmah & Karim, 2018; Kartika & Yazidah, 2019) found that students had many errors in mathematical proof. Students have not been able to make direct definitions and proofs. Furthermore, according to Hodiyanto and Susiaty (2018) a good mathematical proof ability can develop higher-order thinking skills. Furthermore, Firmasari and Sulaiman (2019) stated that the ability to prove mathematically became the basis for several mathematics courses. In line with this, the results of research from Herizal (2020) state that the experience factor is the strongest factor for students in influencing mathematical proof abilities.

Pre-service teacher have difficulty in learning the theory of probability including: difficulty in determining the sample space and Space events, difficult to work on the problem of the complement of events, difficult in solving the problem of compound events and difficulty in working on the problem in the form of a story (Dayat & Limbong, 2012). The Theory of Probability material is one of the meteri in mathematics lessons that have not been mastered by Pre service teacher, one of the causes is the lack of application or context (Zulkardi, 2011). The assumption that mathematics is considered difficult is also expressed by Devlin (2012) sugests that many Pre service teacher have difficulty in math ranging from high school to college level. In his scientific oration stated at the university level he admitted that the ability of pre service teacher is still weak (Patahuddin, 2010).

To be able to send pre service teacher to achieve mathematical learning objectives that have the ability of mathematical proof, then management or mathematical learning approach designed towards the target in order to develop the process of mathematical proof can be done using Three Zone Learning concept, that is Zone Proximal Development (ZPD), Zone Promotion Action (ZPA) and Zone Free Movement (ZFM). Zone of Free Movement (ZFM) is a zone created by teachers to provide space for students to be free to think and do and Zone of Promotion Action (ZPA) is all things given by teachers to students to promote, (Geiger, Anderson & Hurrel, 2017; Hammond & Alotaibi, 2017; Iffah, 2017). This is because three zones learning concept in which there are learning activities that are able to construct student knowledge. Using A Zone Theory analysis of identity formation in mathematics teacher education to investigate identity formation in a pair of mathematics teacher educators – one a mathematician and the other a mathematics educator – who collaborated to develop new approaches to teacher education that integrate content and pedagogy (Merrilyn and Anne, 2019). Learning design is based on the results of various studies and theories with the aim of improving the quality of learning. In review Revisiting Vygotsky’s Concept of Zone of Proximal
Development (ZPD): Towards a Stage of Proximity suggests that the idea of ZPD is an indisputable fact and is a significant issue that gave birth to a new zone theory, namely the Zone of Proximal Development (ZPD) into the Zone of Free Movement (ZFM) (Shokouhi and Shakouri, 2015). Pre service teacher learning or development process is determined by a wide variety of interrelated factors such as ZPD, ZPA, ZFM and is useful for analyzing the extent to which teachers can adopt new teaching practices, Goos (2013). The results of research related to Three zone learning is conducted (Blanton et al, 2005; Brown, 2006; Galligan, 2008; Goos 2013; Hussain et al, 2011; Rahardi, 2011; Smith, 2011; Shabani 2012) in his research adapt three zone learning with socio-cultural. Still proceeding from research based on three zone learning is with (Agyei, 2013; Beninson and Goos, 2013; Handal et al., 2013; Santosa, 2013; Waren et al., 2014) in his research found an illusionary Zone outside the concept of ZFM and ZPA. (Hammond & Alotaibi, 2017; Iffah et al., 2017; Quaicoe & Pata, 2015; Shokouhi & Shakouri, 2015) suggests that identify pseudo-promoting Action in ZPD, ZPA and ZFM. (Geiger, et al., 2017; Jacobs & Usher, 2018) explain the contribution through learning based on three zone learning. Description of three zone learning can be described as follows:

**Dynamic, Interdependent learning community – interactively generates the environment in which learning develops**

![Three Zone Learning Diagram](image)

Picture 1. Three Zone Learning (Goos, 2013)

The urgency of this research is to comprehensively and essentially examine the ability of mathematical proof in solving a mathematical problem in learning through the concept of Three Zone Learning, the novelty and inovation this research by combining Zone Proximal Development, Zone Promotion Action and Zone Free Movement. The next step is to analyze the increase in students' mathematical proofs when viewed from a learning based on the concept of Three Zone Learning.
METHOD

Type of Research

Type of this research is quantitative, which is a research design is a quasi-experimental group and a control group in other words pretest – posttest control group design (Fraenkel, 2012).

Time and Place of Research

The research was carried out in semester 5 of the 2020/2021 academic year, and the place of research is University of Siliwangi Tasikmalaya.

Subjects of Research

The sample subjects were pre service teacher in semester 5, The total number of samples selected was as many as 82 people with the number of experimental group 42 pre service teacher and the control group as many as 40 pre service teacher.

Research Procedure

This research uses a design a quasi – experimental, it consists of two groups, the experimental group and the control group. The experimental group was treated using learning with the concept of three zone learning, and the control group was not treated. Furthermore, both groups were given an pre test and at the end of the learning was given a post test of mathematical proof.

Data analysis instruments and techniques

The data analyzed in this study are data derived from quantitative data obtained from the test data Early Mathematical abilities (EMA), pre test and post test of the ability of mathematical proof. Data processing first of all the data is analyzed by calculating the normalized gain value to identify the improvement of the initial test and the final test of mathematical proof ability. Next, the data is analyzed inferentially to answer the whole of the proposed hypothesis. the next analysis is carried out first of which perform prerequisite testing or parametric assumption testing, namely normality test, n – gain data variant homoginity test from both groups. For testing the normality of data from both groups using Kolmogorov-Smirnov test this is because the number of each group is less than 50. To test the homoginity of N – gain data variant using Levene test. Last in concluding the hypothesis used statistical analysis both parametric and non parametric.

RESULT AND DISCUSSION

Descriptive analysis of EMA data and statistical analysis of the test of differences in mean EMA scores of students between the experimental and control groups. Table 1. The following is the result of data processing which was analyzed descriptively on the initial mathematical abilities of students from two groups, namely the experimental group with Three Zone Learning and the control group, namely conventional learning.
Table 1. Descriptive Statistics of EMA data by Learning and Categories of EMA

<table>
<thead>
<tr>
<th>Category of EMA</th>
<th>Statistics</th>
<th>Modeling</th>
<th>Overall (experimental and control Classes)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>TZL</td>
<td>CL</td>
</tr>
<tr>
<td>Upper</td>
<td>η</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>76,11</td>
<td>76,67</td>
</tr>
<tr>
<td></td>
<td>Standard deviation</td>
<td>4,36</td>
<td>3,95</td>
</tr>
<tr>
<td></td>
<td></td>
<td>22</td>
<td>76,36</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4,09</td>
</tr>
<tr>
<td>Midle</td>
<td>η</td>
<td>21</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>50,16</td>
<td>48,65</td>
</tr>
<tr>
<td></td>
<td>Standard deviation</td>
<td>7,59</td>
<td>8,14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40</td>
<td>49,44</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>7,79</td>
</tr>
<tr>
<td>Lower</td>
<td>η</td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>25,68</td>
<td>26,46</td>
</tr>
<tr>
<td></td>
<td>Standard deviation</td>
<td>4,98</td>
<td>4,49</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>26,11</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4,61</td>
</tr>
<tr>
<td>Totally</td>
<td>η</td>
<td>42</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>52,33</td>
<td>48,56</td>
</tr>
<tr>
<td></td>
<td>Standard deviation</td>
<td>19,02</td>
<td>19,82</td>
</tr>
<tr>
<td></td>
<td></td>
<td>82</td>
<td>50,49</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>19,39</td>
</tr>
</tbody>
</table>

Three Zone Learning (TZL) = Experimental Group
Conventional Learning (CL) = Control Group

The overall total for the mean and standard deviation can be seen in Table 1, which shows that the results are not far apart, so that the experimental group and the control group can and are feasible to apply different treatments. Still in Table 1. Based on the standard deviation, students are grouped based on the upper, middle and lower categories.

The total number of students in this study amounted to 82 people can be seen in Table 3 which is divided into two experimental groups as many as 42 people and a control group as many as 40 people, with the top category for the experimental class as many as 12 people and the control group as many as 10 people with a total category above as many as 22 people or 26.83%. For the middle category in the group The experimental group was 21 people and the control group was 19 people with a total of 40 people in the middle category or 48.78%, and the last in the lower category of the experimental group there were 9 people and the control group 11 people with a total of 20 people 24.39%. It can be seen that in total the category data based on EMA are balanced and not far apart.

Test the difference in the mean of EMA data by category and total learning model. The results of the average difference test based on the upper, middle, lower and total categories of the two models show that for all upper, middle, lower categories and in total there is no significant difference between the average EMA data categorized as learning models.
Overall description of the increase in pre service teacher proof as discussed above has not shown a significant difference when viewed from various factors.

Inferential analysis of students' Mathematical Proofing Ability (MPA) data was processed to determine the increase in students' mathematical proofing abilities, namely from the normalized gain value of students' mathematical proofing abilities. Test of Differences in Average Student MPA Improvement Based on Three Zone Learning (TZL) and CL Modeling. The following are the results of the student MPA improvement test based on the learning model. Table 2. Results of Testing the Differences in Average Student MPA Improvement Based on Learning Modeling Through the Mann - Whitney test

<table>
<thead>
<tr>
<th>MPA Test</th>
<th>Statistic test</th>
<th>Modeling</th>
<th>TZL</th>
<th>CL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enhancement</td>
<td>Mann – Whitney U</td>
<td>639,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Z</td>
<td>-1,950</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Asymp Sig.(2-Tailed)</td>
<td>0,051</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Decision</td>
<td>H&lt;sub&gt;0&lt;/sub&gt; Accepted</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It can be seen in Table 6 that the results show that there is no difference in the average increase in MPA based on the learning model. Next, testing the differences in student MPA improvement based on the TZL and CL modeling can be done through the t-test. The following is the result of the difference in the average increase in student MPA based on the learning model:
Table 3. Results of Testing Differences in Average Student MPA Improvement Based on Learning Modeling Through Test – t

<table>
<thead>
<tr>
<th>MPA Test Enhancement</th>
<th>Statistic Test</th>
<th>Modeling</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t</td>
<td>2,184</td>
</tr>
<tr>
<td></td>
<td>df</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>Sig.</td>
<td>0,032</td>
</tr>
<tr>
<td></td>
<td>Decision</td>
<td>H₀ Rejected</td>
</tr>
</tbody>
</table>

The test results in Table 3 show that there are differences in the average increase in MPA based on the learning model. Testing the Differences in MPA Improvement Based on EMA category. The next discussion is to test and find out the average difference between the two groups, namely TZL and CL, this test is carried out to increase MPA based on the upper, middle, lower and total EMA categories. Next is the test of the difference in the average increase of the two groups, namely TZL and CL based on the EMA category.

Table 4. Results of Testing Differences in Student MPA Improvement Based on Upper, Middle, Lower EMA Categories and Learning Modeling Through the Mann - Whitney test

<table>
<thead>
<tr>
<th>MPA Test Enhancement</th>
<th>EMA Category</th>
<th>Statistic Test</th>
<th>Modeling</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Upper</td>
<td>Mann – Whitney U</td>
<td>48,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Z</td>
<td>-0,871</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Asymp Sig.(2-Tailed)</td>
<td>0,384</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Keputusan</td>
<td>H₀ Accepted</td>
</tr>
<tr>
<td></td>
<td>Midle</td>
<td>Mann – Whitney U</td>
<td>180,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Z</td>
<td>-0,556</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Asymp Sig.(2-Tailed)</td>
<td>0,578</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Keputusan</td>
<td>H₀ Accepted</td>
</tr>
<tr>
<td></td>
<td>Lower</td>
<td>Mann – Whitney U</td>
<td>20,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Z</td>
<td>-2,480</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Asymp Sig.(2-Tailed)</td>
<td>0,013</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Decision</td>
<td>H₀ Rejected</td>
</tr>
</tbody>
</table>

It can be seen in Table 4 that the decision from the Mann – Whitney U statistical test on the upper and middle MPA with the TZL and CL models is accepted, this means that the average data on increasing student CAR and TZL and CL modeling based on the upper and middle EMA categories have no difference. Meanwhile, the lower MPA with the TZL and CL models were rejected, this means that the average data on the increase in student CAR and TZL and CL models based on the lower EMA category are different.

For normalized gain based on EMA in total and learning modeling, the next test uses the Kruskal Wallis test.
The decisions from Table 5 are the same or in other words there is no difference. The next test is the One-Way Anova Post Hoc test, this test intends to see differences in student MPA normalized gain data for each EMA category.

### Table 6. Post Hoc Test Results Differences in MPA Normalized Gain Based on EMA Category

<table>
<thead>
<tr>
<th>Modeling</th>
<th>Statistic Test</th>
<th>EMA Category (I)</th>
<th>EMA Category (J)</th>
<th>Mean Difference (I – J)</th>
<th>Sig.</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>TZL and CL</td>
<td><em>Bonferroni-Test</em></td>
<td>Upper Midle Lower</td>
<td>0,012” 0,028”</td>
<td>0,035 0,000</td>
<td>There is a difference There is a difference</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Midle Upper Lower</td>
<td>-0,012” 0,016”</td>
<td>0,035 0,003</td>
<td>There is a difference There is a difference</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Lower Upper Midle</td>
<td>-0,028” -0,016”</td>
<td>0,000 0,003</td>
<td>There is a difference There is a difference</td>
<td></td>
</tr>
</tbody>
</table>

Table 6 Post hoc Bonferroni test the difference in average normalized MPA gain based on the upper, middle and lower total EMA categories for categories between upper and middle EMA, between upper and lower EMA, between middle and upper EMA, between middle and lower EMA, between EMA bottom and top, between bottom and middle EMA shows that Sig. < which means that there is a significant difference in the improvement of student MPA between the EMA categories.

Indicator of achievement of the aspect of mathematical proof ability. In general, the difficulty faced by students is that they cannot show and validate proofs, this is due to a lack of understanding of the concept of several discrete special distributions.

The following is a research that is relevant to previous research, namely: Blanton, Westbrook, and Carter (2005) this research explores the use of three zone learning as a way to interpret the zone of proximal development. The results of the study showed that teachers who provide space for students to be free to think and do within the scope of ZFM and continue with the promotion process to students provide better improvement achievements. Brown (2006) in his research apply through the concept of three Zone Learning in technology-based learning. It is mentioned that there is a balance of teachers and students in teaching through ZPD, ZFM and ZPA. Galligan (2008) in his investigation three zone learning is very suitable in setting the teaching and learning process that involves students and teachers so that students' abilities can develop quickly. Smith (2011) in his research concluded that
students’ actions are influenced by transformation that is able to increase the ability to self-confidence to be greater, able to review students’ mistakes appropriately, increased commitment to investigation, able to change from hamabatan to opportunity. Rahardi (2011) examines the learning activities of teachers and students through three Learning zone describes the student zone formed through the guidance of his teacher and concluded that students become active in learning.

CONCLUSION

Based on table 6 about Post Hoc Test Results Differences in MPA Normalized Gain Based on EMA . Category that the EMA level which has a positive value of 0.012 in the column (IU) with the upper category (I) and the middle category EMA level (J) means that the average MPA score of students from the upper category EMA level (I ) is better than the EMA level in the middle category (J), then also for the EMA level which has a positive value of 0.028 in the column (IU) with the upper category (I) and the lower category EMA level (J) which means that the average MPA score students from the upper category EMA level (I) are better than the lower category EMA level (J). As for the EMA level which has a positive value of 0.016 in the column (IU) with the middle category (I) and the lower category EMA level (J) it means that the average MPA score of students from the middle category (I) EMA level is better than the EMA level. lower category (J). From the results in Table 10, it is shown that for the EMA level with the upper, middle and lower categories, the effect is greater in increasing student

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